

THERMODYNAMICS PROCESS

a) Constant volume process

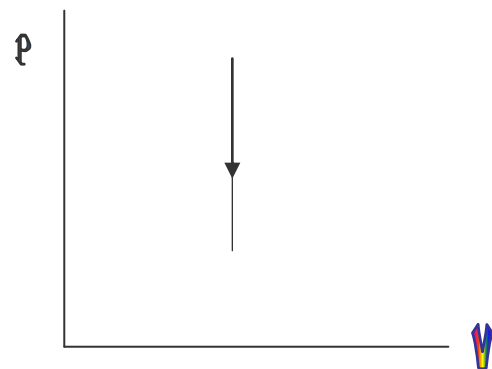
In this process

$$V = \text{constant}$$

$$dV = 0$$

$$W = \int_1^2 P dV = 0$$

$$Q - W = \Delta U$$



and $W=0$ for constant volume Fig(1.1) constant volume process

$$Q = \Delta U$$

$$Q = m * (u_2 - u_1)$$

u_2 internal energy at the final state (kJ/kg)

u_1 internal energy at the initial state (kJ/kg)

For ideal gas

$$\Delta U = m * C_v * \Delta T$$

$$Q = m * C_v * (T_2 - T_1)$$

Example 1.1

A rigid tank contains air at 500kPa and 150°C. As a result of the surrounding, the temperature and pressure inside the tank drop to 65°C and 400kPa, respectively. Determine the work done during this process.

Solution: Given $T_1=150^\circ\text{C}$ and $P_1=500\text{kPa}$

$T_2=65^\circ\text{C}$ and $P_2=400\text{kPa}$ with no change in

volume because the tank is rigid.

$V=\text{constant}$ and $dv=0$ and so $W=0$

b) Constant Pressure Process

$$P = \text{constant}$$

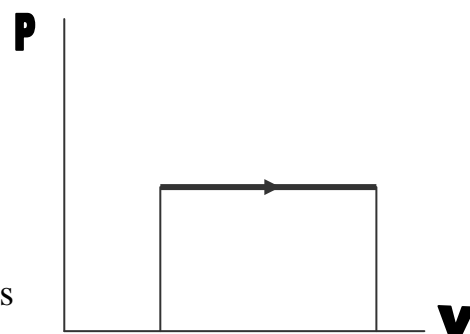
$$P = P_1 = P_2$$

$$W = \int_1^2 P dV = P(V_2 - V_1)$$

in this process the work of ideal gas is

$$W = P(V_2 - V_1)$$

$$W = mR(T_2 - T_1)$$



Fig(1.2) constant pressure process

and for vapor

$$W = mP(v_2 - v_1)$$

$$Q - W = \Delta U$$

$$Q = \Delta U + W$$

$$Q = \Delta H$$

H=enthalpy (KJ)

$$Q = m * (h_2 - h_1)$$

h_2 internal energy at the final state (kj/kg)

h_1 internal energy at the initial state (kj/kg)

For ideal gas

$$\Delta U = m * C_p * \Delta T$$

$$Q = m C_p * (T_2 - T_1)$$

Example 1.2

Five kilograms of saturated vapor water at 1Mpa is contained in a cylinder fitted with a movable piston. This system is now heated at constant pressure until the temperature of the steam is 300°C. Calculate the work done by the steam during the process.

Solution: Given sat vapor water

$$m = 5 \text{ kg} \quad P_1 = P_2 = 1 \text{ MPa}$$

$$T_2 = 300^\circ \text{C}$$

From the saturated water table

$$v_1 = v_{g \text{ at } 1 \text{ MPa}} = 0.19444 \text{ m}^3 / \text{kg}$$

$$T_{sat} = 179.91^\circ \text{C}$$

the second state is a super heated vapor because $T_2 > T_{sat}$

$$v_2 = v_{at \text{ 1 MPa } 300^\circ \text{C}} = 0.2579 \text{ m}^3 / \text{kg}$$

the process is constant pressure expansion

$$W = mP(v_2 - v_1)$$

$$W = 5 \text{ kg} \times 1000 \text{ kPa} \times (0.2579 \text{ m}^3 / \text{kg} - 0.19444 \text{ m}^3 / \text{kg})$$

$$W = 317.3 \text{ kJ}$$

H.W Q=?

c) Hyperbolic Process

In this process

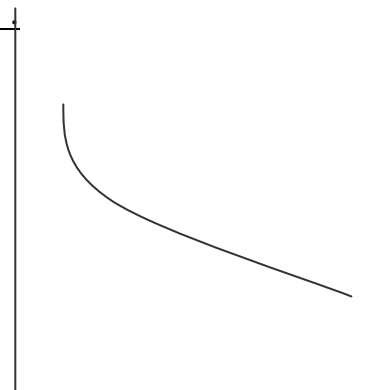


Fig.hyperbolic process

$$PV = \text{Const.} = c \quad \rightarrow \quad P = \frac{c}{V}$$

$$W = \int_1^2 p dV = \int_1^2 \frac{c}{V} dV = c \int_1^2 \frac{dV}{V} = c \ln \frac{V_2}{V_1}$$

$$W = P_1 V_1 \ln \frac{V_2}{V_1} = P_2 V_2 \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{P_1}{P_2} = P_2 V_2 \ln \frac{P_1}{P_2}$$

this process are called isothermal process for ideal gas(only), because for ideal gases when $PV = \text{const.}$ so $T = \text{const.}$

where $Q=W$ for ideal gas

Example 1.3

One tenth kg of saturated vapor water is at 2MPa is compressed in hyperbolic process to a pressure of 4MPa . Find the final temperature of the water and the work done.

Solution: Given $m=0.1\text{kg}$ $P_1=2\text{MPa}$ sat water vapor

$P_2=4\text{MPa}$ and the process is $PV=\text{constant}$

At the first state $v_1 = v_g \text{ at } 2\text{MPa} = 0.09963\text{m}^3 / \text{kg}$

$$P_2 v_2 = P_1 v_1 \quad \text{OR} \quad v_2 = v_1 \frac{P_1}{P_2} = 0.09963\text{m}^3 / \text{kg} \times \frac{2\text{MPa}}{4\text{MPa}} = 0.04982\text{m}^3 / \text{kg}$$

the sat. volume at 4MPa $v_g = 0.04978\text{m}^3 / \text{kg}$

it is found that $v_2 > v_g \text{ at } 4\text{MPa}$ so the state is superheated vapor

to find the temperature by using the superheated water table and interpolation as follows

<u>T °C</u>	<u>v m³ / kg</u>
250.4	0.04978
	0.04982
<u>275.0</u>	<u>0.05457</u>

$$T = 250.4 + \frac{(0.04982 - 0.04978)}{(0.05457 - 0.04978)} (275 - 250.4) = 250.6^\circ \text{C}$$

and the work can be calculated by

$$W = m P_1 v_1 \ln \frac{P_1}{P_2} = 0.1 \times 2000 \times 0.09963 \ln \frac{2}{4} = -13.812\text{kJ}$$

d) Isothermal Process (Constant Temperature Process)

This process can be discussed separately for ideal gas and vapor

1- Ideal gas

when the temperature is constant ($T = \text{constant}$) and from the ideal gas equation of state, with no change in the mass ($PV = \text{constant}$), the process becomes hyperbolic process and

$$W = P_1V_1 \ln \frac{V_2}{V_1} = P_2V_2 \ln \frac{V_2}{V_1} = P_1V_1 \ln \frac{P_1}{P_2} = P_2V_2 \ln \frac{P_1}{P_2}$$

$$PV = mRT$$

$$W = mRT \ln \frac{V_2}{V_1} = mRT \ln \frac{P_1}{P_2}$$

Example 1.4

One kilogram of air at 500°C is expanded isothermally from a pressure of 2MPa to a pressure of 0.5MPa, find the work done by the air.

Solution: Given Air of $m=1\text{kg}$ at $P_1=2\text{MPa}$ $P_2=0.5\text{MPa}$ $T_1=T_2=T=500^\circ\text{C}$
It is an ideal gas and isothermal process of expansion

$$W = mRT \ln \frac{P_1}{P_2} =$$

$$W = 1\text{kg} \times 0.287 \times (500 + 273.15) \ln \frac{2}{0.5} = 307.61\text{kJ}$$

2-Substance with phase change

(i) Saturated region

in saturated region when the temperature is constant the pressure is also constant because the pressure and temperature are dependent properties $P=f(T)$. Therefore the work in this process is the same to that as in constant pressure process

(ii) Superheated region

In this region the temperature and pressure are not dependent properties ($P \neq f(T)$ only). Therefore the process can be assumed as polytropic process ($PV^n = \text{constant}$)

Example 1.5

0.4kg of saturated liquid water at 120°C is vaporized in piston cylinder device isothermally until the volume of liquid becomes one tenth of the total volume. Find the work done by the system.

Solution: Given sat. liquid water $m=0.4\text{ kg}$ $T=120^\circ\text{C}$ isothermally

$$V_{f2} = \frac{V_2}{10}, \quad V_{g2} = \frac{9V_2}{10}$$

as the water is still in the saturated region the expansion is also constant pressure of $P = P_{\text{sat at } 120^\circ\text{C}} = 198.53\text{kPa}$

$$v_f = 0.00106\text{m}^3/\text{kg} \quad v_g = 0.8919\text{m}^3/\text{kg} \quad v_1 = v_f = 0.00106\text{m}^3/\text{kg}$$

$$V_{f2} = m_f v_f = \frac{V_2}{10} = \frac{m v_2}{10} \rightarrow v_2 = 10 \frac{m_f}{m} v_f = 10(1-x)v_f$$

$$V_{g2} = m_g v_g = \frac{9V_2}{10} = \frac{9m v_2}{10} \rightarrow v_2 = \frac{10}{9} \frac{m_g}{m} v_g = \frac{10}{9} x v_g$$

$$v_2 = 10(1-x)v_f = \frac{10}{9} x v_g \rightarrow x = \frac{v_f}{\frac{1}{9}v_g + v_f} = \frac{0.00106}{\frac{0.8919}{9} + 0.00106} = 0.0106$$

$$v_2 = v_f + x(v_g - v_f) = 0.00106 + 0.0106(0.8919 - 0.00106) = 0.0105\text{m}^3/\text{kg} \text{ or}$$

$$v_2 = \frac{10}{9} x v_g = \frac{10}{9} \times 0.0106 \times 0.8919 = 0.0105\text{m}^3/\text{kg}$$

$$W = mP(v_2 - v_1) = 0.4 \times 198.53 \times (0.0105 - 0.00106) = 0.75\text{kJ}$$

e) Polytropic Process

During expansion and compression processes of real gases, pressure and volume are often related by $(PV^n = c)$ where n, and c are constants. A process of this kind is called a polytropic process.

$$W = \int_1^2 P dV$$

$$PV^n = c \rightarrow P = \frac{c}{V^n} = cV^{-n}$$

$$W = \int_1^2 cV^{-n} dV = \frac{cV^{-n+1}}{1-n} \Big|_1^2 = \frac{PV^n V^{1-n}}{1-n} \Big|_1^2 = \frac{PV}{1-n} \Big|_1^2$$

$$W = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

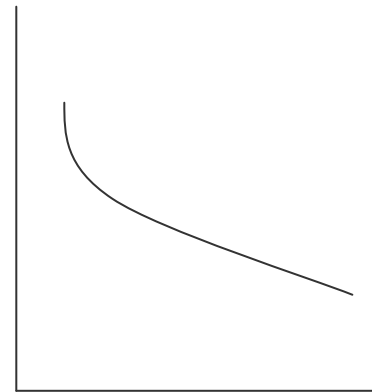


Fig. polytropic process

For change phase substance the polytropic process

$$W = \frac{m(P_2 v_2 - P_1 v_1)}{1-n} \text{ where } v \text{ is the specific volume}$$

the ideal gas polytropic process can be written as

$$W = \frac{P_2V_2 - P_1V_1}{1-n} \quad \text{or} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$$

$$W = \frac{mR(T_2 - T_1)}{1-n} \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

for ideal gas in polytropic process we can drive the following relation:

$$P_1V_1 = mRT_1 \quad \text{and} \quad P_2V_2 = mRT_2$$

$$P_1V_1^n = C \quad \text{and} \quad P_2V_2^n = C$$

$$P_1V_1^n = P_2V_2^n$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

for ideal gas only

Example 1.6

Carbon dioxide with mass of 5kg at 100kPa pressure and 300K temperature is compressed polytropically according to the law $PV^{1.32}=C$ until the pressure of 500kPa. Find (a) initial and final volume (b) the final temperature (c) the work done

Solution: Given CO_2 gas $m = 5\text{kg}$ $P_1 = 100\text{kPa}$ $T_1 = 300^\circ\text{C}$

$P_2 = 500\text{kPa}$ for CO_2 the gas constant $R=0.2968\text{kJ/kg.K}$

$$V_1 = \frac{mRT_1}{P_1} = \frac{5 \times 0.1889 \times 300}{100} = 2.8335\text{m}^3$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 2.8335 \times \left(\frac{100}{500}\right)^{\frac{1}{1.32}} = 0.8371\text{m}^3$$

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 300 \times \left(\frac{500}{100}\right)^{\frac{1.32-1}{1.32}} = 443.2\text{K}$$

$$W = \frac{mR(T_2 - T_1)}{1-n} = \frac{5 \times 0.1889(443.2 - 300)}{1-1.32} = 422.5\text{kJ} \quad \text{or}$$

$$W = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{500 \times 0.8371 - 100 \times 2.8335}{1-1.32} = -422.5$$

for a cyclic process, the initial and final state are identical, and therefore $\Delta U = U_2 - U_1 = 0$. then the first law relation for a cycle simplifies to

$$Q - W = 0$$

Example 1.8

A rigid vessel of $.1\text{m}^3$ volume contains refrigerant-12 5% liquid and 95% vapor by volume at 24°C . the vessel is heated until the refrigerant exit as saturated vapor. Find (a) the initial pressure in the vessel (b) the mass of Refrigerant-12, (c) the final pressure and temperature, and (d) heat transfer occurs during the process:

Solution: Given $V=0.1\text{m}^3$ 10% liquid 90% vapor $T_1=24^\circ\text{C}$ second state is sat. vapor.

$$V_f = 0.05V = 0.05 \times 0.1 = 0.005\text{m}^3, \quad V_g = 0.95V = 0.95 \times 0.1 = 0.095\text{m}^3$$

and from the sat. R-12 table we find that the following properties at 24°C

$$P_{sat} = 634.05\text{kPa}, \quad v_f = 0.0007607\text{m}^3/\text{kg}, \quad v_g = 0.02759\text{m}^3/\text{kg},$$

$$u_f = 58.25\text{kJ}/\text{kg}, \quad u_g = 179.85\text{kJ}/\text{kg}$$

(a) The initial pressure: because the initial state is saturated mixture at 24°C , then $P_1 = P_{sat \text{ at } 24^\circ\text{C}} = 634.05\text{ kPa}$

(b) The mass of R-12 in the vessel

$$m_f = \frac{V_f}{v_f} = \frac{0.005}{0.0007607} = 6.573\text{kg}, \quad m_g = \frac{V_g}{v_g} = \frac{0.095}{0.02759} = 3.443\text{kg}$$

$$m = m_f + m_g = 6.573 + 3.443 = 10.016\text{kg}$$

(c) the final pressure and temperature of the R-12 in the vessel: the

final state is saturated vapor with $v_2 = \frac{V}{m} = \frac{0.1}{10.016} = .01\text{m}^3/\text{kg}$

and at the second state $v_g = v_2 = 0.01\text{m}^3/\text{kg}$

from the pressure table we find the following data and using extrapolation we can find the data at the $v_g = 0.01\text{m}^3/\text{kg}$

P kPa	T °C	$v_f \text{ m}^3/\text{kg}$	$v_g \text{ m}^3/\text{kg}$	$u_f \text{ kJ}/\text{kg}$	$u_g \text{ kJ}/\text{kg}$
1400	56.09	0.0008448	0.01222	90.28	191.11
1600	62.19	0.0008660	0.01054	96.80	192.95
1664.4	64.15	0.0008728	0.01	98.90	193.54

So $P_2 = 1664.4\text{kPa}$, $T_2 = 64.15^\circ\text{C}$

(d) The heat transfer: because there is no change in volume so $W=0$

$$Q = m(u_2 - u_1)$$

$$u_1 = u_f + x(u_g - u_f) = 58.25 + 0.344(179.85 - 58.25) = 100.08\text{kJ}/\text{kg}$$

$$u_2 = u_{g \text{ at second state}} = 193.54\text{kJ}/\text{kg}$$

$$Q = 10.016 \times (193.54 - 100.08) = 2940.9\text{kJ}$$

Example 1.9

A rigid insulated tank of 0.5m^3 , contains 5 kg of water at 100°C . An electric heater is passing through the tank with a voltage of 200V and a current of 5A for 30 minutes. Find the final state of water.

Solution: Given $V=0.5\text{m}^3$, rigid, insulated $Q=0$, $m=5\text{kg}$ water, $T=100^\circ\text{C}$, electric heater $V=200\text{Volt}$, $I=5\text{A}$, $\text{time}=30\text{minutes}=1800\text{sec}$.

The energy equation can be written as:

$$Q - W_e - W_b = \Delta U$$

where $Q=0$ for insulated tank, $W_b=\text{boundary work}=0$ rigid tank

$W_e = \text{electric work} = V \times I \times \text{time} / 1000 = 200 \times 5 \times 1800 / 1000 = 1800\text{kJ}$

This work is negative because it is done in the system.

$$-W_e = m\Delta u$$

$$-(-1800) = 5\Delta u$$

$$\Delta u = 360\text{kJ}$$

from the first state $T=100^\circ\text{C}$, and $v_1 = \frac{V}{m} = \frac{0.5}{5} = 0.1\text{m}^3/\text{kg}$

and it is shown that the state is saturated mixture because $v_f < v_1 < v_g$

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{0.1 - 0.001044}{1.6729 - 0.001044} = 0.06$$

$$u_1 = u_f + x_1 u_{fg} = 418.94 + 0.06 \times 2087.6 = 542.5\text{kJ/kg}$$

$$u_2 = u_1 + \Delta u = 542.5 + 360 = 902.5\text{kJ/kg}$$

$$v_2 = v_1 = 0.1\text{m}^3/\text{kg}$$

it is shown from the values of the internal energy and specific volume that the water is still saturated mixture. And by trial and error we can get the temperature or pressure.

$$T_2 = 134.9^\circ\text{C}, P_2 = 312.3\text{kPa}, x = 0.1696 = 16.96\%$$

Example 1.11

A piston cylinder device contains water at 300kPa, and 250°C with a volume of 0.4m^3 . If the weight of the piston is required a pressure of 300kPa to rise it. The heat is transfer until the water become saturated mixture with quality of 80%. (a) prove that the heat transfer in a constant pressure process equal to the change in enthalpy. (b) the work done (c) heat transfer during the process.

Solution: Given $P_1 = 300\text{kPa}$, $V_1 = 0.4\text{m}^3$, $T_1 = 250^\circ\text{C}$ constant pressure

process. $x_2 = 0.8$

$$(a) \text{ for constant pressure process } W = P(V_2 - V_1) = PV_2 - PV_1$$

the energy equation for closed system $Q = W + \Delta U$

$$Q = PV_2 - PV_1 + U_2 - U_1 = (P_2V_2 + U_2) - (P_1V_1 + U_1)$$

$$Q = H_2 - H_1 = \Delta H$$

(b) For $P_1 = 300\text{kPa}$, $T_1 = 250^\circ\text{C}$, from superheated water table

$$v_1 = 0.7964\text{m}^3/\text{kg}, \quad u_1 = 2728.7\text{kJ}/\text{kg}, \quad h_1 = 2967.6\text{kJ}/\text{kg}$$

at state 2 $P_2 = 300\text{kPa}$, $x_2 = 0.8$

$$v_2 = v_f + x(v_g - v_f) = 0.001073 + 0.8(0.6058 - 0.001073) = 0.4849\text{m}^3/\text{kg}$$

$$u_2 = u_f + xu_{fg} = 561.15 + 0.8 \times 1982.4 = 2147.07\text{kJ}/\text{kg}$$

$$h_2 = h_f + xh_{fg} = 561.47 + 0.8 \times 2163.8 = 2292.51\text{kJ}/\text{kg}$$

$$m = \frac{V_1}{v_1} = \frac{0.4}{0.7964} = 0.5023\text{m}^3/\text{kg}$$

$$W = mP(v_2 - v_1) = 0.5023 \times 300 \times (0.4849 - 0.7964) = -46.94\text{kJ}$$

(c) the heat transfer: it can be calculated by two ways,

$$(i) \quad Q = W + m\Delta u = -46.94 + 0.5023 \times (2147.07 - 2728.7) = -339.1\text{kJ}$$

$$(ii) \quad Q = m\Delta h = 0.5023 \times (2292.5 - 2967.6) = -339.1\text{kJ}$$

Example 1.12

Air is in a rigid tank of volume 1m^3 at initial pressure of 500kPa and temperature of 300K . It is heated to a final temperature of 700K . (a) find the final pressure in the tank, (b) the change in internal energy and enthalpy, (c) heat transfer to the system.

Solution: Given $P_1=500\text{kPa}$, $T_1=300\text{K}$, The tank is rigid with volume of $V=1\text{m}^3$, $T_2=700\text{K}$, from table of ideal gas properties. $R=0.287\text{kJ}/\text{kg}\cdot\text{K}$, $C_p=1.005\text{kJ}/\text{kg}\cdot\text{K}$, $C_v=0.718\text{kJ}/\text{kg}\cdot\text{K}$

(a)

$$P_2 = P_1 \frac{T_2}{T_1} = 500 \times \frac{700}{300} = 1166.67\text{kPa}$$

$$(b) \quad m = \frac{P_1 V}{RT_1} = \frac{500 \times 1}{0.287 \times 300} = 5.807\text{kg}$$

$$\Delta U = mC_v(T_2 - T_1) = 5.807 \times 0.718 \times (700 - 300) = 1667.77\text{kJ}$$

$$\Delta H = mC_p(T_2 - T_1) = 5.807 \times 1.005 \times (700 - 300) = 2334.414\text{kJ}$$

(c) because the tank is rigid, $W=0$

$$Q = \Delta U = 1667.77\text{kJ}$$

Example 1.13

Nitrogen gas is heated in a piston-cylinder device from 30°C to 120°C at constant pressure of 200kPa . The mass of nitrogen in the system is 0.2kg . Find the work done and heat transfer during the process.

Solution: Given N_2 , $m=0.2\text{kg}$, $T_1=30^\circ\text{C}=303\text{K}$, $T_2=120^\circ\text{C}=393\text{K}$ at $P=200\text{kPa}$, from table $R=0.2968\text{kJ}/\text{kg}\cdot\text{K}$, $C_p=1.039\text{kJ}/\text{kg}\cdot\text{K}$

$$W = mR\Delta T = 0.2 \times 0.2968 \times (120 - 30) = 5.3424\text{kJ}$$

$$Q = mC_p\Delta T = 0.2 \times 1.039 \times (120 - 30) = 18.702\text{kJ}$$

Example 1.14

A rigid tank of volume 0.5m^3 contains saturated water mixture with quality of 50% at 120°C is heated until its temperature becomes 200°C . Find (a) final state of the water and pressure, (b) heat transfer (c) change of entropy.

Solution: The given rigid tank $V=0.5\text{m}^3$ sat water $T_1=120^\circ\text{C}$, $x_1=0.5$
The final temperature is $T_2=200^\circ\text{C}$

At $T_1=120^\circ\text{C}$ and from sat. water table it is found that

$$v_{f1} = 0.001060 \text{ m}^3 / \text{kg} \quad v_{g1} = 0.8919 \text{ m}^3 / \text{kg}$$

$$u_{f1} = 503.5 \text{ kJ} / \text{kg} \quad u_{fg1} = 2025.8 \text{ kJ} / \text{kg}$$

$$s_{f1} = 1.5276 \text{ kJ} / \text{kg.K} \quad s_{fg1} = 5.602 \text{ kJ} / \text{kg.K}$$

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 0.00106 + 0.5 \times (0.8919 - 0.00106) = 0.44648 \text{ m}^3 / \text{kg}$$

$$m = \frac{V}{v_1} = \frac{0.5}{0.44648} = 1.12 \text{ kg}$$

$$u_1 = u_{f1} + x_1 u_{fg1} = 503.5 + 0.5 \times 2025.8 = 1516.4 \text{ kJ} / \text{kg}$$

$$s_1 = s_{f1} + x_1 s_{fg1} = 1.5276 + 0.5 \times 5.602 = 4.3286 \text{ kJ} / \text{kg.K}$$

because the tank is rigid $v_1 = v_2$

$$v_2 = 0.44648 \text{ m}^3 / \text{kg} \text{ at } T_2 = 200^\circ\text{C}$$

It is found that the second state is super heated steam because $v_2 > v_g$ at the temperature 200°C . the pressure is between 0.4MPa and 0.5MPa

And by using the interpolation as follows

P MPa	v m ³ /kg	u kJ/kg	s kJ/kg.K
0.4	0.5342	2646.8	7.1706
.48	0.44648	2643.67	7.0812
0.5	0.4249	2642.9	7.0592

(a) the final pressure is $P_2=480\text{kPa}$

(b) because the tank is rigid

$$Q = \Delta U = m\Delta u = 1.12(2646.67 - 1516.4) = 1265.9 \text{ kJ}$$

$$\Delta S = m\Delta s = m(s_2 - s_1) = 1.12(7.0812 - 4.3286) = 3.083$$

Example 1.15

Piston cylinder contains 2kg of steam at a pressure of 200kPa and quality of 75% is heated with constant pressure until it becomes dry saturated vapor. Find (a) the work done (b) the heat transfer and (c) the change in entropy.

Solution: the given sat. water mixture at $P_1=200\text{kPa}$ and $x_1=0.75$ $m=2\text{kg}$, the final state is sat. vapor at the same pressure of 200kPa.

$$(a) v_1 = v_f + x_1 v_{fg} \quad \text{and} \quad v_2 = v_g \quad \text{at} \quad P = 200 \text{ kPa}$$

$$W = P(V_2 - V_1) = Pm(v_2 - v_1) = Pm(v_g - (v_f + x_1 v_{fg}))$$

$$W = Pm(v_{fg} - x_1 v_{fg}) Pm(1 - x_1)(v_g - v_f) = 200 \times 2 \times (1 - 0.75)(0.8857 - 0.001061)$$

$$W = 88.464 \text{ kJ}$$

$$(b) h_1 = h_f + x_1 h_{fg} \quad \text{and} \quad h_2 = h_g$$

$$Q = m\Delta h = m(h_2 - h_1) = m(1 - x_1)h_{fg} = 2 \times (1 - 0.75) \times 2209.1 = 1104.55 \text{ kJ}$$

$$(c) s_1 = s_f + x_1 s_{fg} \quad \text{and} \quad s_2 = s_g$$

$$\Delta S = m(s_2 - s_1) = m(1 - x_1)s_{fg} = 2 \times (1 - 0.75) \times 5.597 = 2.7985 \text{ kJ / K}$$

Example 1.16

One kg of steam at a pressure of 700kPa and quality of 80% is expanded hyperbolically to a pressure of 150kPa. Determine (a) the final state of the vapor and (b) change in entropy

Solution: the given $m=1\text{kg}$, $P_1=700\text{kPa}$, $x_1=0.8$, $P_2=150\text{kPa}$

The Process is hyperbolically $P_1 V_1 = P_2 V_2$

At $P_1=700\text{kPa}$, $x_1=0.8$

$$v_1 = v_f + x_1(v_g - v_f) = 0.001108 + 0.8 \times (0.02729 - 0.001108) = 0.21854 \text{ m}^3 / \text{kg}$$

$$s_1 = s_f + x_1 s_{fg} = 1.9922 + 0.8 \times 4.7158 = 5.7648 \text{ kJ / kg.K}$$

$$v_2 = \frac{P_1 v_1}{P_2} = \frac{700 \times 0.21854}{150} = 1.01985 \text{ m}^3 / \text{kg}$$

$$v_2 < v_g \quad \text{at} \quad P_2 = 150 \text{ kPa}$$

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{1.0198 - 0.001053}{1.1593 - 0.001053} = 0.8796 = 87.96\%$$

$$s_2 = s_f + x_2 s_{fg} = 1.4336 + 0.8796 \times 5.7897 = 6.526 \text{ kJ / kg.K}$$

$$\Delta S = m(s_2 - s_1) = 1 \times (6.526 - 5.7648) = 0.7612 \text{ kJ / K}$$

Example 1.17

Piston cylinder device contains 0.5kg of refrigerant-12 at 20°C and quality of 40%. The refrigerant is expanded isothermally until its final pressure becomes 240kPa. Find (1) the change in entropy (2) heat transfer (3) the work done.

Solution: the given R-12 $T_1=T_2=20^\circ\text{C}$, $x_1=0.4$, $P_2=240\text{kPa}$

At the initial state

$$u_1 = u_f + x_1 u_{fg} = 54.44 + 0.4 \times (178.32 - 54.44) = 103.992 \text{ kJ / kg}$$

$$s_1 = s_f + x_1 s_{fg} = 0.2078 + .4 \times (0.6884 - 0.2078) = 0.4 \text{ kJ / kg.K}$$

at the final state

$$u_2 = 182.53 \text{ kJ / kg} \quad s_2 = 0.7624 \text{ kJ / kg.K}$$

$$(1) \Delta S = m(s_2 - s_1) = 0.5 \times (0.7624 - 0.4) = 0.1812 \text{ kJ / K}$$

(2) the process is isothermal so $T = \text{constant}$

$$Q = \int T ds = T \int ds = T \Delta S = (20 + 273) \times 0.1812 = 53.092 \text{ kJ}$$

$$(3) W = Q - \Delta U = 53.092 - 0.5 \times (182.53 - 103.992) = 13.787 \text{ kJ}$$

Example 1.18

0.05m³ of air at a pressure of 800kPa and temperature 20°C expands to eight times its original volume and the final temperature after expansion is 25°C. Calculate the work done and heat transfer

Solution: the givens air $V_1 = 0.084 \text{ m}^3$, $P_1 = 1250 \text{ kPa}$, $T_1 = T_2 = 537^\circ \text{C} = 810 \text{ K}$,
 $V_2 = 0.336 \text{ m}^3$

$$(i) P_2 = P_1 \frac{V_1}{V_2} = 1250 \times \frac{0.084}{0.336} = 312.5 \text{ kPa}$$

$$(ii) W = P_1 V_1 \ln \frac{V_2}{V_1} = 1250 \times 0.084 \times \ln \frac{0.336}{0.084} = 145.561 \text{ kJ}$$

$$(iii) Q = W = 145.56 \text{ kJ}$$

Example 1.19

A volume of 0.14m³ of air at 100kPa and 90°C is compressed to 0.014m³ according to $PV^{1.3} = \text{const}$. Heat is then added at a constant volume until the pressure is 6600kPa. Determine :

(1) heat exchange with the cylinder walls during compression,
and

Solution: given Air $V_1 = 0.14 \text{ m}^3$, $P_1 = 100 \text{ kPa}$, $T_1 = 90^\circ \text{C}$, $V_2 = 0.014 \text{ m}^3$,

$$P_3 = 6600 \text{ kPa}, P_1 V_1^{1.3} = P_2 V_2^{1.3}, V_2 = V_3$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 0.14}{0.287 \times (90 + 273)} = 0.1855 \text{ kg}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{n-1} = 363 \left(\frac{0.14}{0.014} \right)^{1.3-1} = 724.28 \text{ K}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^n = 100 \left(\frac{0.14}{0.014} \right)^{1.3} = 1995 \text{ kPa}$$

$$W_{12} = \frac{mR(T_2 - T_1)}{1-n} = \frac{0.1855 \times 0.287 \times (724.28 - 363)}{1-1.3} = -64.11 \text{ kJ}$$

$$\Delta U_{12} = mCv(T_2 - T_1) = 0.1855 \times 0.718 \times (724.28 - 363) = 48.12 \text{ kJ}$$

$$Q_{12} = W_{12} + \Delta U_{12} = -64.11 + 48.12 = 15.99 \text{ kJ}$$

6.5 Adiabatic (Isentropic) Process:

The adiabatic process is a process in which no heat transfer to or from the process. It means that

$$\delta Q = 0$$

$$\delta Q = TdS$$

The temperature is in absolute value so it is not equal zero.

Then

$$dS = 0$$

$$S = \text{constant}$$

therefore, the adiabatic process is called a constant entropy or (isentropic process).

For ideal gas from the third equation in which

$$\Delta S = m \left(C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1} \right)$$

for adiabatic process $\Delta S = 0$

$$0 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1}$$

$$\ln \frac{P_2}{P_1} = -k \ln \frac{V_2}{V_1} = \ln \left(\frac{V_1}{V_2} \right)^k$$

by exponential the equation we get

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^k$$

or

$$P_1 V_1^k = P_2 V_2^k$$

then for adiabatic process of an ideal gas the relation between pressure and volume is

$$PV^k = \text{Const.}$$

For this process we can show that

$$\frac{P_2}{P_1} = \frac{P_{r1}}{P_{r2}}$$

This process is look like the polytropic process except $n=k$ the specific heat ratio (also called the adiabatic index). Then the work for this process is

$$W = \frac{P_2 V_2 - P_1 V_1}{1-k} = \frac{mR(T_2 - T_1)}{1-k} = -mC_v(T_2 - T_1) = -(U_2 - U_1) = -\Delta U$$

The heat transfer from a system during a process is defined by the relation

$$Q = \int_1^2 TdS \quad \text{or} \quad q = \int_1^2 Tds$$

This relation is look like the work done during the process relation with a pressure and differential form of volume. So the area under the curve on a T-s diagram represents the heat transfer during the process as shown in fig.6.3. On this diagram the constant temperature process is a horizontal line, and the constant entropy process as a vertical line as

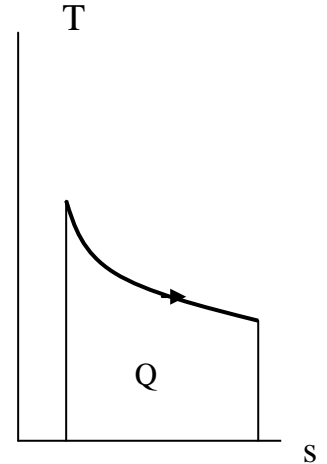


Fig.
T-s diagram

shown on the fig.6.4. From this we denoted that at constant entropy line the area under the curve is equal to zero. So through this process there is no heat transfer.

The Carnot cycle of heat engine is shown on the fig.. the analysis of this cycle is as follows;

$$s_1 = s_4, \quad s_2 = s_3 \quad \text{and} \quad T_1 = T_2 = T_H, \quad T_3 = T_4 = T_L$$

$$q_H = T_H(s_2 - s_1)$$

$$q_L = T_L(s_3 - s_4) = T_L(s_2 - s_1)$$

$$w = q_H - q_L = (T_H - T_L)(s_2 - s_1)$$

the efficiency of the cycle

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{T_L(s_2 - s_1)}{T_H(s_2 - s_1)} = 1 - \frac{T_L}{T_H}$$

also the work can be represented by the area rounded by the cycle .

Example 1.20

Saturated vapor refrigerant-12 at -10°C , is compressed adiabatically in piston and cylinder device to a pressure of 600kPa, calculate the final temperature and the work done per unit mass.

Solution: given R-12 saturated vapor at $T_1 = -10^\circ\text{C}$, adiabatically $s_1 = s_2$, $Q = 0$, $P_2 = 600\text{kPa}$

$$s_1 = s_g = 0.7019 \text{ kJ/kg.K}, \quad u_1 = u_g = 166.39 \text{ kJ/kg}$$

$$s_2 = s_1 = 0.7019 \text{ kJ/kg.K} \quad \text{at} \quad P_2 = 600 \text{ kPa}$$

by using interpolation

$$T_2 = 22 + \frac{0.7019 - 0.6878}{0.7068 - 0.6878}(30 - 22) = 27.94^\circ\text{C}$$

$$u_2 = 179.09 + \frac{0.7019 - 0.6878}{0.7068 - 0.6878}(184.01 - 179.09) = 182.74 \text{ kJ/kg}$$

$$w = -\Delta u = -(182.74 - 166.39) = -16.35 \text{ kJ/kg}$$

Example 1.21

5kg of Air at 1000K and 2MPa is expanded adiabatically in a closed system to the temperature of 600K, find the final pressure and volume of the air, and the work done.

Solution: given air $m = 5\text{kg}$, $T_1 = 1000\text{K}$, $P_2 = 2\text{MPa} = 2000\text{kPa}$,

$T_2 = 600\text{K}$, expansion is adiabatically, ($s_2 = s_1$), or ($P_1 V_1^K = P_2 V_2^K$)

$$V_1 = \frac{mRT_1}{P_1} = \frac{5 \times 0.287 \times 1000}{2000} = 0.7175 \text{ m}^3$$

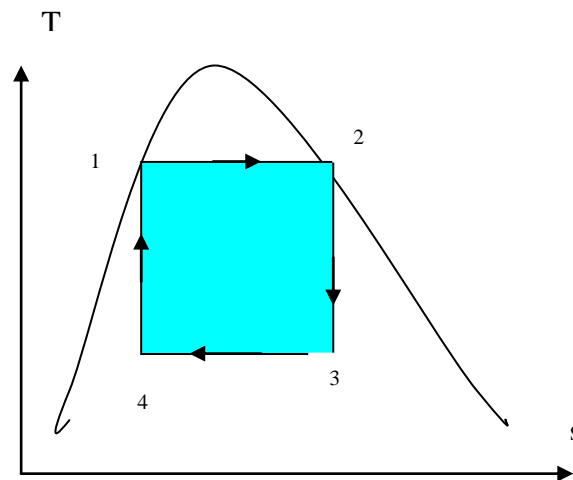
$$V_2 = V_1 \left(\frac{T_1}{T_2} \right)^{1/k-1} = 0.7175 \left(\frac{1000}{600} \right)^{1/(1.4-1)} = 2.573 \text{ m}^3$$

$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{K/k-1} = 2000 \left(\frac{600}{1000} \right)^{1.4/(1.4-1)} = 334.626 \text{ kPa}$$

$$W = \frac{mR(T_2 - T_1)}{1 - K} = \frac{5 \times 0.287 \times (600 - 1000)}{1 - 1.4} = 1435 \text{ kJ}$$

The T-s diagram can also be plotted for the pure substances such as water vapor, in which it indicates the region of the saturation lines. The constant pressure and temperature process between the saturation line is a horizontal line, and the heat added to the vapor is equal to the latent heat of vaporization (h_{fg}).

$$q = h_{fg}$$



Carnot cycle on T-s diagram with respect to saturation lines

and because this process is constant temperature so

$$q = T s_{fg}$$

from these two relations we find that

$$s_{fg} = \frac{h_{fg}}{T}$$

where the temperature is in absolute value.

Fig. 6.6 shows the Carnot cycle on a T-s diagram for a pure substance with respect to the saturation lines.

$$s_3 = s_2, \quad s_4 = s_1$$

$$q_H = T_H (s_2 - s_1)$$

$$q_L = T_L (s_2 - s_1)$$

$$s_3 = s_f + x_3 s_{fg}, \quad s_4 = s_f + x_4 s_{fg}, \quad \text{at } T_L, \text{ or } P_L$$

PROBLEM

1.1 Calculate the work done and heat transfer of 2kg of air, when it is heated at constant volume from 100kPa to 400kPa

1.2 Air occupies 0.084m^3 at 1.25MPa and 537°C . It is expanded at a constant temperature to a final volume of 0.336m^3 . Calculate :the pressure at the end of expansion, (ii) work done during expansion (iii) heat transfer to the air

1.3 A vessel having a volume of 5 m^3 contains 0.05 m^3 of saturated liquid water and 4.95 m^3 of saturated water vapor at 0.1 MPa. Heat is transferred until the vessel is filled with saturated vapor. Determine the heat transfer for this process.

1.4 Determine the missing property (P , T , or x) and v for water at each of the following states:

a. $T = 300^\circ\text{C}$, $u = 2780\text{ kJ/kg}$

b. $P = 2000\text{ kPa}$, $u = 2000\text{ kJ/kg}$

1.5 A piston/cylinder contains 2 kg water at 20AAoEEAAC with volume 0.1 mAA3EEAA. By mistake someone locks the piston preventing it from moving while we heat the water to saturated vapor. Find the final temperature, volume and the process work.

1.6 Saturated vapor R-410A at 0AAoEEAAC in a rigid tank is cooled to -20AAoEEAAC. Find the specific heat transfer.

1.7 A rigid tank holds 0.75 kg water at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

1.8 A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.

1.9 A rigid 10-L vessel initially contains a mixture of liquid water and vapor at 100°C with 12.3 percent quality. The mixture is then heated until its temperature is 180°C. Calculate the heat transfer required for this process.

1.10 Nitrogen in a rigid vessel is cooled by rejecting 100 kJ/kg of heat. Determine the internal energy change of the nitrogen, in kJ/kg.

1.11 A rigid tank contains 1.5 kg of R-134a at 40°C, 500 kPa. The tank is placed in a refrigerator that brings it to -20°C. Find the process heat transfer

1.12 Two kg water at 120°C with a quality of 25% has its temperature raised 20°C in a constant volume process. What are the heat transfer and w_k in the process.

1.13 A mass of 200 g of saturated liquid water is completely vaporized at a constant pressure of 100 kPa. Determine (a) the work done and (b) the amount of energy transferred to the water.

1.14 A piston–cylinder device initially contains 0.4 m³ of air at 100 kPa and 80°C. The air is now compressed to 0.1 m³ in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.

1.15 A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. (a) Show that for a closed system the boundary work W_b and the change in internal energy ΔU in the first-law relation can be combined into one term, ΔH , for a constant-pressure process. (b) Determine the final temperature of the steam.

1.15 Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass

1.17 A piston–cylinder device contains 0.005 m³ of liquid water and 0.9 m³ of water vapor in equilibrium at 600 kPa. Heat is transferred at constant pressure until the temperature reaches 200°C.

- (a) What is the initial temperature of the water?
- (b) Determine the total mass of the water.
- (c) Calculate the final volume.

1.18 Water initially at 200 kPa and 300°C is contained in a piston–cylinder device fitted with stops. The water is allowed to cool at constant pressure until it exists as a saturated vapor and the piston rests on the stops. Then the water continues to cool until the pressure is 100 kPa. Determine (a) the work done and (b) the amount of energy transferred

1.19 One kilogram of water fills a 150-L rigid container at an initial pressure of 2 MPa. The container is then cooled to 40°C. Determine the initial temperature and the final pressure of the water.