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chapter 5

Beams

5.1 INTRODUCTION

Beams are structural members that support transverse loads and are therefore subjected primarily to flexure, or bending. If a substantial amount of axial load is also present, the member is referred to as a *beam–column* (beam–columns are considered in Chapter 6). Although some degree of axial load will be present in any structural member, in many practical situations this effect is negligible and the member can be treated as a beam. Beams are usually thought of as being oriented horizontally and subjected to vertical loads, but that is not necessarily the case. A structural member is considered to be a beam if it is loaded so as to cause bending.

Commonly used cross-sectional shapes include the W, S, and M shapes. Channel shapes are sometimes used, as are beams built up from plates, in the form of I or box shapes. For reasons to be discussed later, doubly symmetric shapes such as the standard rolled W, M, and S shapes are the most efficient.

Coverage of beams in the AISC Specification is spread over two chapters: Chapter F, "Design of Members for Flexure," and Chapter G, "Design of Members for Shear." Several categories of beams are covered in the Specification; in this book, we cover the most common cases in the present chapter, and we cover a special case, plate girders, in Chapter 10.

Figure 5.1 shows two types of beam cross sections; a hot-rolled doubly symmetric I shape and a welded doubly symmetric built-up I shape. The hot-rolled I shape is the one most commonly used for beams. Welded shapes usually fall into the category classified as plate girders.

For flexure (shear will be covered later), the required and available strengths are moments. For load and resistance factor design (LRFD), Equation 2.6 can be written as

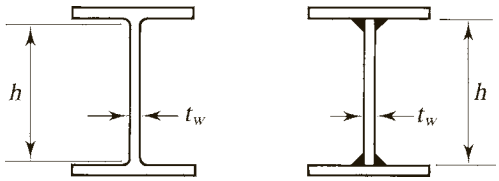
$$M_u \leq \phi_b M_n \tag{5.1}$$

where

M_u = required moment strength = maximum moment caused by the controlling load combination from ASCE 7
ϕ_b = resistance factor for bending (flexure) = 0.90
M_n = nominal moment strength

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FIGURE 5.1



The right-hand side of Equation 5.1 is the design strength, sometimes called the *design moment.*

For allowable strength design (ASD), Equation 2.7 can be written as

$$M_a \leq \frac{M_n}{\Omega_b} \tag{5.2}$$

where

M_a = required moment strength = maximum moment corresponding to the controlling load combination from ASCE 7
Ω_b = safety factor for bending = 1.67

Equation 5.2 can also be written as

$$M_a \leq \frac{M_n}{1.67} = 0.6M_n$$

Dividing both sides by the elastic section modulus S (which will be reviewed in the next section), we get an equation for *allowable stress design:*

$$\frac{M_a}{S} \leq \frac{0.6M_n}{S}$$

or

$$f_b \leq F_b$$

where

f_b = maximum computed bending stress
F_b = allowable bending stress

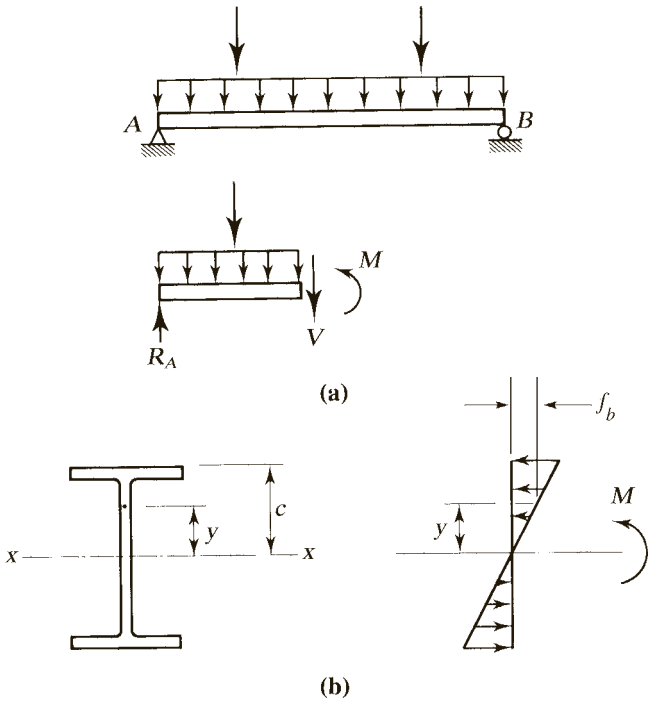
5.2 BENDING STRESS AND THE PLASTIC MOMENT

To be able to determine the nominal moment strength M_n, we must first examine the behavior of beams throughout the full range of loading, from very small loads to the point of collapse. Consider the beam shown in Figure 5.2a, which is oriented so that bending is about the major principal axis (for an I shape, it will be the x–x axis). For a linear elastic material and small deformations, the distribution of bending stress will be as shown in Figure 5.2b, with the stress assumed to be uniform across the width of the beam. (Shear is considered separately in Section 5.8.) From elementary mechanics of materials, the stress at any point can be found from the flexure formula:

$$f_b = \frac{My}{I_x} \tag{5.3}$$

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FIGURE 5.2



where M is the bending moment at the cross section under consideration, y is the perpendicular distance from the neutral plane to the point of interest, and I_x is the moment of inertia of the area of the cross section with respect to the neutral axis. For a homogeneous material, the neutral axis coincides with the centroidal axis. Equation 5.3 is based on the assumption of a linear distribution of strains from top to bottom, which in turn is based on the assumption that cross sections that are plane before bending remain plane after bending. In addition, the beam cross section must have a vertical axis of symmetry, and the loads must be in the longitudinal plane containing this axis. Beams that do not satisfy these criteria are considered in Section 5.15. The maximum stress will occur at the extreme fiber, where y is maximum. Thus there are two maxima: maximum compressive stress in the top fiber and maximum tensile stress in the bottom fiber. If the neutral axis is an axis of symmetry, these two stresses will be equal in magnitude. For maximum stress, Equation 5.3 takes the form:

$$f_{\max} = \frac{Mc}{I_x} = \frac{M}{I_x/c} = \frac{M}{S_x} \tag{5.4}$$

where c is the perpendicular distance from the neutral axis to the extreme fiber, and S_x is the elastic section modulus of the cross section. For any cross-sectional shape, the section modulus will be a constant. For an unsymmetrical cross section, S_x will have two values: one for the top extreme fiber and one for the bottom. Values of S_x for standard rolled shapes are tabulated in the dimensions and properties tables in the *Manual.*

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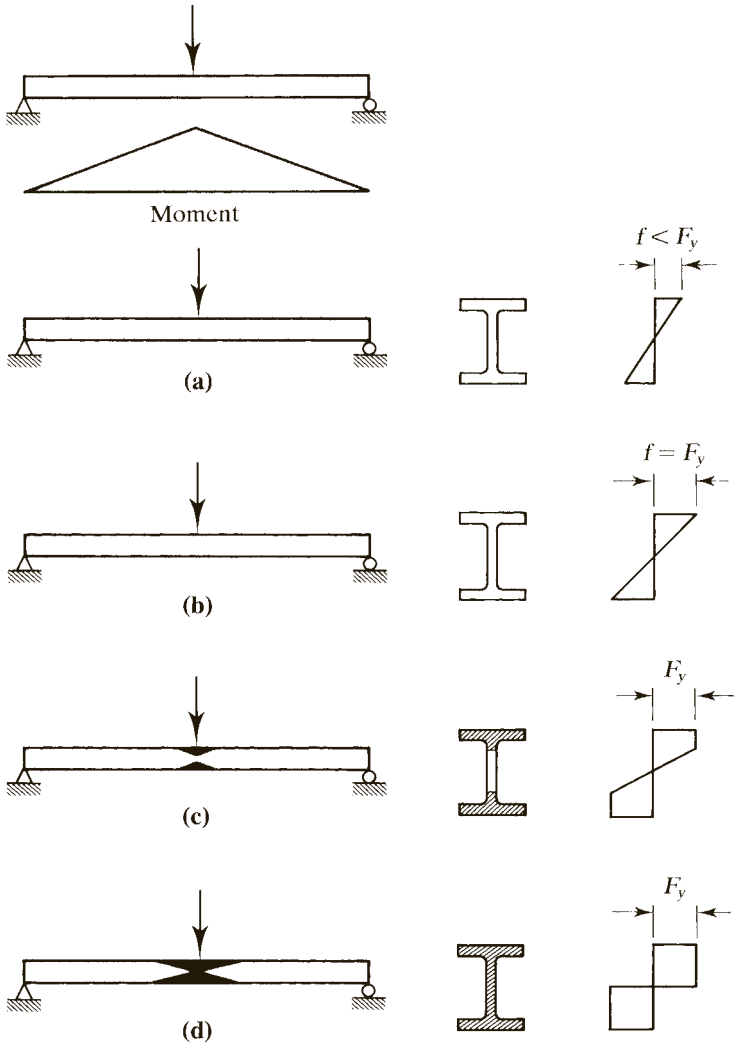
Equations 5.3 and 5.4 are valid as long as the loads are small enough that the material remains within its linear elastic range. For structural steel, this means that the stress f_{max} must not exceed F_y and that the bending moment must not exceed

$$M_y = F_y S_x$$

where M_y is the bending moment that brings the beam to the point of yielding.

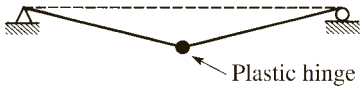
In Figure 5.3, a simply supported beam with a concentrated load at midspan is shown at successive stages of loading. Once yielding begins, the distribution of stress on the cross section will no longer be linear, and yielding will progress from the extreme fiber toward the neutral axis. At the same time, the yielded region will extend longitudinally from the center of the beam as the bending moment reaches M_y at more locations. These yielded regions are indicated by the dark areas in Figure 5.3c and d.

FIGURE 5.3



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FIGURE 5.4



In Figure 5.3b, yielding has just begun. In Figure 5.3c, the yielding has progressed into the web, and in Figure 5.3d the entire cross section has yielded. The additional moment required to bring the beam from stage b to stage d is 10 to 20% of the yield moment, M_y, for W shapes. When stage d has been reached, any further increase in the load will cause collapse, since all elements of the cross section have reached the yield plateau of the stress–strain curve and unrestricted plastic flow will occur. A *plastic hinge* is said to have formed at the center of the beam, and this hinge along with the actual hinges at the ends of the beam constitute an unstable mechanism. During plastic collapse, the mechanism motion will be as shown in Figure 5.4. Structural analysis based on a consideration of collapse mechanisms is called *plastic analysis*. An introduction to plastic analysis and design is presented in the Appendix of this book.

The plastic moment capacity, which is the moment required to form the plastic hinge, can easily be computed from a consideration of the corresponding stress distribution. In Figure 5.5, the compressive and tensile stress resultants are shown, where A_c is the cross-sectional area subjected to compression, and A_t is the area in tension. These are the areas above and below the plastic neutral axis, which is not necessarily the same as the elastic neutral axis. From equilibrium of forces,

$$\begin{aligned} C &= T \\ A_c F_y &= A_t F_y \\ A_c &= A_t \end{aligned}$$

Thus the plastic neutral axis divides the cross section into two equal areas. For shapes that are symmetrical about the axis of bending, the elastic and plastic neutral axes are the same. The plastic moment, M_p, is the resisting couple formed by the two equal and opposite forces, or

$$M_p = F_y(A_c)a = F_y(A_t)a = F_y\left(\frac{A}{2}\right)a = F_y Z$$

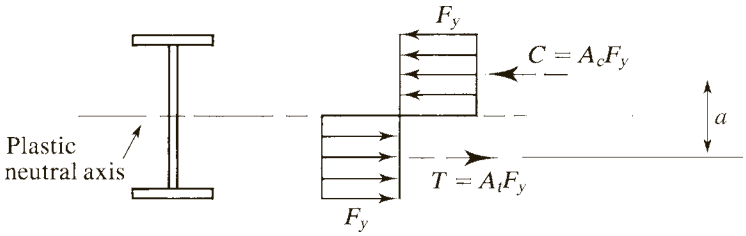
where

A = total cross-sectional area

a = distance between the centroids of the two half-areas

$Z = \left(\frac{A}{2}\right)a$ = plastic section modulus

FIGURE 5.5

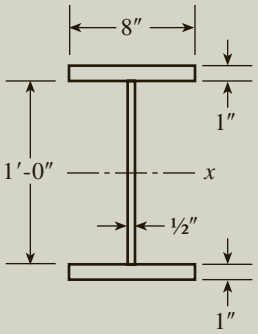


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Example 5.1

For the built-up shape shown in Figure 5.6, determine (a) the elastic section modulus S and the yield moment M_y and (b) the plastic section modulus Z and the plastic moment M_p. Bending is about the x-axis, and the steel is A572 Grade 50.

FIGURE 5.6



Solution **a.** Because of symmetry, the elastic neutral axis (the x-axis) is located at middepth of the cross section (the location of the centroid). The moment of inertia of the cross section can be found by using the parallel axis theorem, and the results of the calculations are summarized in Table 5.1.

TABLE 5.1

Component	$\bar{I}$	A	d	$I + Ad^2$
Flange	0.6667	8	6.5	338.7
Flange	0.6667	8	6.5	338.7
Web	72	—	—	72.0
Sum				749.4

The elastic section modulus is

$$S = \frac{I}{c} = \frac{749.4}{1 + (12/2)} = \frac{749.4}{7} = 107 \text{ in.}^3$$

and the yield moment is

$$M_y = F_y S = 50(107) = 5350 \text{ in.-kips} = 446 \text{ ft-kips}$$

Answer $S = 107$ in.^{3} and $M_y = 446$ ft-kips.

b. Because this shape is symmetrical about the x-axis, this axis divides the cross section into equal areas and is therefore the plastic neutral axis. The centroid

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of the top half-area can be found by the principle of moments. Taking moments about the x-axis (the neutral axis of the entire cross section) and tabulating the computations in Table 5.2, we get

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{61}{11} = 5.545 \text{ in.}$$

TABLE 5.2

Component	A	y	Ay
Flange	8	6.5	52
Web	3	3	9
Sum	11		61

FIGURE 5.7

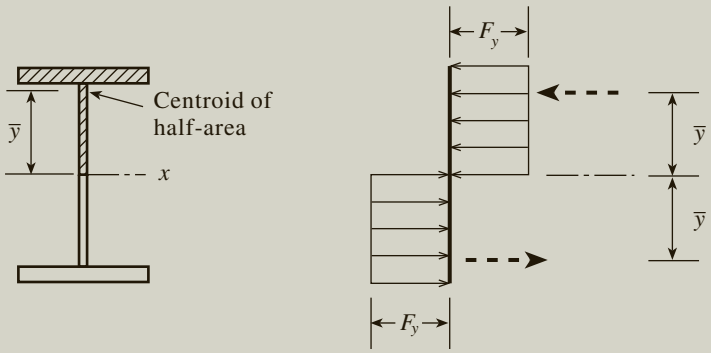


Figure 5.7 shows that the moment arm of the internal resisting couple is

$$a = 2\bar{y} = 2(5.545) = 11.09 \text{ in.}$$

and that the plastic section modulus is

$$\left(\frac{A}{2}\right)a = 11(11.09) = 122 \text{ in.}^3$$

The plastic moment is

$$M_p = F_y Z = 50(122) = 6100 \text{ in.-kips} = 508 \text{ ft-kips}$$

Answer $Z = 122 \text{ in.}^3$ and $M_p = 508$ ft-kips.

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Example 5.2

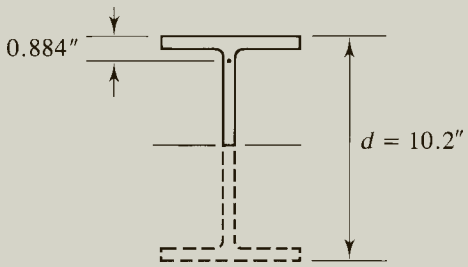
Compute the plastic moment, M_p, for a W10 × 60 of A992 steel.

Solution From the dimensions and properties tables in Part 1 of the *Manual,*

$$A = 17.7 \text{ in.}^2$$
$$\frac{A}{2} = \frac{17.7}{2} = 8.85 \text{ in.}^2$$

The centroid of the half-area can be found in the tables for WT shapes, which are cut from W shapes. The relevant shape here is the WT5 × 30, and the distance from the outside face of the flange to the centroid is 0.884 inch, as shown in Figure 5.8.

FIGURE 5.8



$$a = d - 2(0.884) = 10.2 - 2(0.884) = 8.432 \text{ in.}$$
$$Z = \left(\frac{A}{2}\right)a = 8.85(8.432) = 74.62 \text{ in.}^3$$

This result, when rounded to three significant figures, is the same as the value given in the dimensions and properties tables.

Answer $M_p = F_y Z = 50(74.62) = 3731$ in.-kips $= 311$ ft-kips.

5.3 STABILITY

If a beam can be counted on to remain stable up to the fully plastic condition, the nominal moment strength can be taken as the plastic moment capacity; that is,

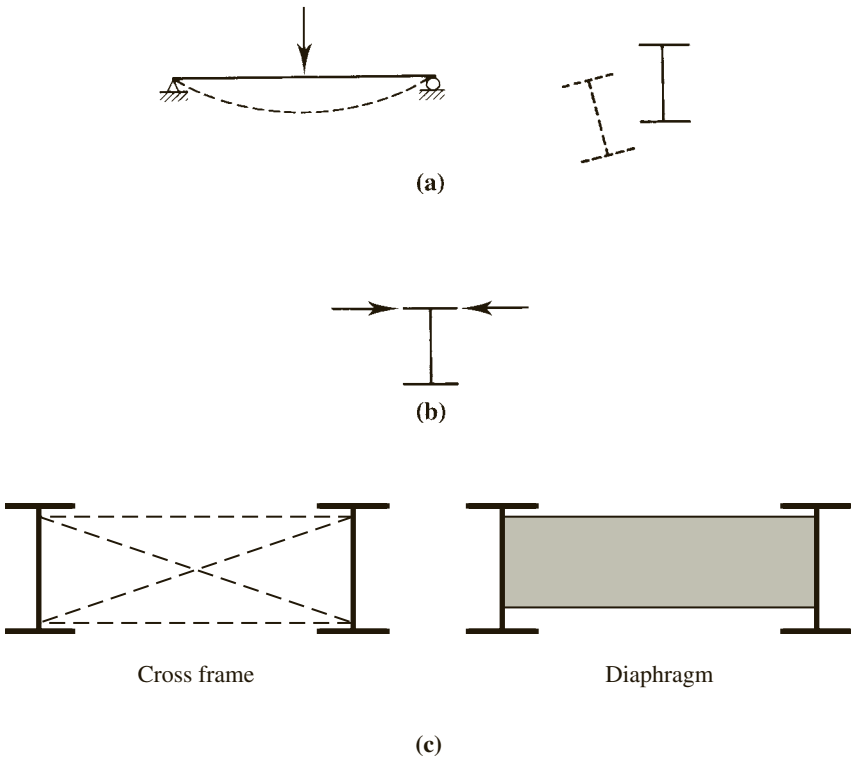
$$M_n = M_p$$

Otherwise, M_n will be less than M_p.

As with a compression member, instability can be in an overall sense or it can be local. Overall buckling is illustrated in Figure 5.9a. When a beam bends, the

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FIGURE 5.9

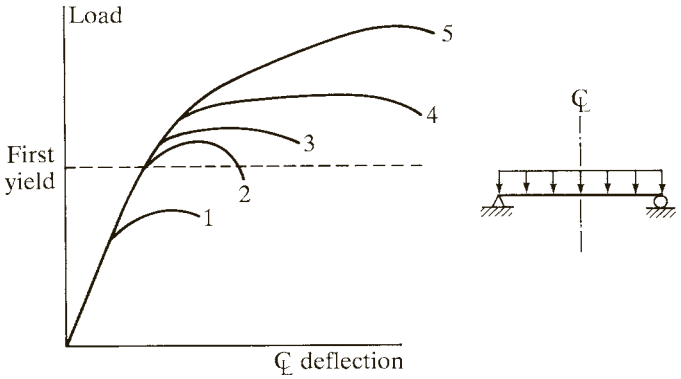


compression region (above the neutral axis) is analogous to a column, and in a manner similar to a column, it will buckle if the member is slender enough. Unlike a column, however, the compression portion of the cross section is restrained by the tension portion, and the outward deflection (flexural buckling) is accompanied by twisting (torsion). This form of instability is called *lateral-torsional buckling* (LTB). Lateral-torsional buckling can be prevented by bracing the beam against twisting at sufficiently close intervals. This can be accomplished with either of two types of stability bracing: lateral bracing, illustrated schematically in Figure 5.9b, and torsional bracing, represented in Figure 5.9c. Lateral bracing, which prevents lateral translation, should be applied as close to the compression flange as possible. Torsional bracing prevents twist directly; it can be either nodal or continuous, and it can take the form of either cross frames or diaphragms. The nodal and relative categories were defined in Chapter 4, "Compression Members." Appendix 6 of the AISC Specification gives the strength and stiffness requirements for beam bracing. These provisions are based on the work of Yura (2001). As we shall see, the moment strength depends in part on the unbraced length, which is the distance between points of bracing.

Whether the beam can sustain a moment large enough to bring it to the fully plastic condition also depends on whether the cross-sectional integrity is maintained. This integrity will be lost if one of the compression elements of the cross section buckles. This type of buckling can be either compression flange buckling, called

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FIGURE 5.10



flange local buckling (FLB), or buckling of the compression part of the web, called *web local buckling* (WLB). As discussed in Chapter 4, "Compression Members," whether either type of local buckling occurs will depend on the width-to-thickness ratios of the compression elements of the cross section.

Figure 5.10 further illustrates the effects of local and lateral-torsional buckling. Five separate beams are represented on this graph of load versus central deflection. Curve 1 is the load-deflection curve of a beam that becomes unstable (in any way) and loses its load-carrying capacity before first yield (see Figure 5.3b) is attained. Curves 2 and 3 correspond to beams that can be loaded past first yield but not far enough for the formation of a plastic hinge and the resulting plastic collapse. If plastic collapse can be reached, the load-deflection curve will have the appearance of either curve 4 or curve 5. Curve 4 is for the case of uniform moment over the full length of the beam, and curve 5 is for a beam with a variable bending moment (moment gradient). Safe designs can be achieved with beams corresponding to any of these curves, but curves 1 and 2 represent inefficient use of material.

5.4 CLASSIFICATION OF SHAPES

AISC classifies cross-sectional shapes as compact, noncompact, or slender, depending on the values of the width-to-thickness ratios. For I shapes, the ratio for the projecting flange (an *unstiffened* element) is $b_f/2t_f$, and the ratio for the web (a *stiffened* element) is h/t_w. The classification of shapes is found in Section B4 of the Specification, "Member Properties," in Table B4.1b (Table B4.1a is for compression members). It can be summarized as follows. Let

λ = width-to-thickness ratio
λ_p = upper limit for compact category
λ_r = upper limit for noncompact category

Then

if $\lambda \leq \lambda_p$ and the flange is continuously connected to the web, the shape is compact;
if $\lambda_p < \lambda \leq \lambda_r$, the shape is noncompact; and
if $\lambda > \lambda_r$, the shape is slender.

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TABLE 5.3 Width-to-Thickness Parameters*

Element	λ	λ_p	λ_r
Flange	$\frac{b_f}{2t_f}$	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$
Web	$\frac{h}{t_w}$	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$

*For hot-rolled I shapes in flexure.

The category is based on the worst width-to-thickness ratio of the cross section. For example, if the web is compact and the flange is noncompact, the shape is classified as noncompact. Table 5.3 has been extracted from AISC Table B4.1b and is specialized for hot-rolled I-shaped cross sections.

Table 5.3 also applies to channels, except that λ for the flange is b_f/t_f.

5.5 BENDING STRENGTH OF COMPACT SHAPES

A beam can fail by reaching M_p and becoming fully plastic, or it can fail by

1. lateral-torsional buckling (LTB), either elastically or inelastically;
2. flange local buckling (FLB), elastically or inelastically; or
3. web local buckling (WLB), elastically or inelastically.

If the maximum bending stress is less than the proportional limit when buckling occurs, the failure is said to be *elastic*. Otherwise, it is *inelastic*. (See the related discussion in Section 4.2, "Column Theory.")

For convenience, we first categorize beams as compact, noncompact, or slender, and then determine the moment resistance based on the degree of lateral support. The discussion in this section applies to two types of beams: (1) hot-rolled I shapes bent about the strong axis and loaded in the plane of the weak axis, and (2) channels bent about the strong axis and either loaded through the shear center or restrained against twisting. (The shear center is the point on the cross section through which a transverse load must pass if the beam is to bend without twisting.) Emphasis will be on I shapes. C-shapes are different only in that the width-to-thickness ratio of the flange is b_f/t_f rather than $b_f/2t_f$.

We begin with *compact shapes*, defined as those whose webs are continuously connected to the flanges and that satisfy the following width-to-thickness ratio requirements for the flange and the web:

$$\frac{b_f}{2t_f} \le 0.38\sqrt{\frac{E}{F_y}} \quad \text{and} \quad \frac{h}{t_w} \le 3.76\sqrt{\frac{E}{F_y}}$$

The web criterion is met by all standard I and C shapes listed in the *Manual* for $F_y \le 65$ ksi; therefore, in most cases only the flange ratio needs to be checked (note that built-up welded I shapes can have noncompact or slender webs). Most shapes

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will also satisfy the flange requirement and will therefore be classified as compact. The noncompact shapes are identified in the dimensions and properties table with a footnote (footnote f). Note that compression members have different criteria than flexural members, so a shape could be compact for flexure but slender for compression. As discussed in Chapter 4, shapes with slender compression elements are identified with a footnote (footnote c). If the beam is compact and has continuous lateral support, or if the unbraced length is very short, the nominal moment strength, M_n, is the full plastic moment capacity of the shape, M_p. For members with inadequate lateral support, the moment resistance is limited by the lateral-torsional buckling strength, either inelastic or elastic.

The first category, laterally supported compact beams, is quite common and is the simplest case. For a doubly symmetric, compact I- or C-shaped section bent about its major axis, AISC F2.1 gives the nominal strength as

$$M_n = M_p \qquad \text{(AISC Equation F2-1)}$$

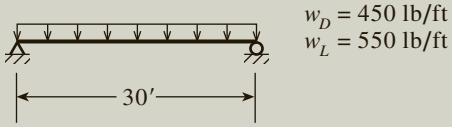
where

$$M_p = F_y Z_x$$

Example 5.3

The beam shown in Figure 5.11 is a W16 × 31 of A992 steel. It supports a reinforced concrete floor slab that provides continuous lateral support of the compression flange. The service dead load is 450 lb/ft. This load is superimposed on the beam; it does not include the weight of the beam itself. The service live load is 550 lb/ft. Does this beam have adequate moment strength?

FIGURE 5.11



Solution First, determine the nominal flexural strength. Check for compactness.

$$\frac{b_f}{2t_f} = 6.28 \qquad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 > 6.28 \quad \therefore \text{ The flange is compact.}$$

$$\frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}} \quad \therefore \text{ The web is compact.}$$

(The web is compact for all shapes in the *Manual* for $F_y \leq 65$ ksi.)

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This shape can also be identified as compact because there is no footnote in the dimensions and properties tables to indicate otherwise. Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(54.0) = 2700 \text{ in.-kips} = 225.0 \text{ ft kips}.$$

Compute the maximum bending moment. The total service dead load, including the weight of the beam, is

$$w_D = 450 + 31 = 481 \text{ lb/ft}$$

For a simply supported, uniformly loaded beam, the maximum bending moment occurs at midspan and is equal to

$$M_{\max} = \frac{1}{8} wL^2$$

where w is the load in units of force per unit length, and L is the span length. Then

$$M_D = \frac{1}{8} w_D L^2 = \frac{1}{8}(0.481)(30)^2 = 54.11 \text{ ft-kips}$$

$$M_L = \frac{1}{8}(0.550)(30)^2 = 61.88 \text{ ft-kips}$$

LRFD solution

The dead load is less than 8 times the live load, so load combination 2 controls:

$$M_u = 1.2M_D + 1.6M_L = 1.2(54.11) + 1.6(61.88) = 164 \text{ ft-kips}.$$

Alternatively, the loads can be factored at the outset:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.481) + 1.6(0.550) = 1.457 \text{ kips/ft}$$

$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(1.457)(30)^2 = 164 \text{ ft-kips}$$

The design strength is

$$\phi_b M_n = 0.90(225.0) = 203 \text{ ft-kips} > 164 \text{ ft-kips} \qquad \text{(OK)}$$

Answer

The design moment is greater than the factored-load moment, so the W16 × 31 is satisfactory.

ASD solution

ASD load combination 2 controls.

$$M_a = M_D + M_L = 54.11 + 61.88 = 116.0 \text{ ft-kips}$$

Alternatively, the loads can be added before the moment is computed:

$$w_a = w_D + w_L = 0.481 + 0.550 = 1.031 \text{ kips/ft}$$

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.031)(30)^2 = 116.0 \text{ ft-kips}$$

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The allowable moment is

$$\frac{M_n}{\Omega_b} = \frac{M_n}{1.67} = 0.6M_n = 0.6(225.0) = 135 \text{ ft-kips} > 116 \text{ ft-kips} \qquad \text{(OK)}$$

Allowable Stress Solution.

The applied stress is

$$f_b = \frac{M_a}{S_x} = \frac{116.0(12)}{47.2} = 29.5 \text{ ksi}$$

The allowable stress is

$$F_b = \frac{0.6M_n}{S_x} = \frac{0.6(225.0)(12)}{47.2} = 34.3 \text{ ksi}$$

Since $f_b < F_b$, the beam has enough strength.

Answer The W16 × 31 is satisfactory.

The allowable stress solution can be simplified if a slight approximation is made. The allowable stress can be written as

$$F_b = \frac{0.6M_n}{S_x} = \frac{0.6F_yZ_x}{S_x}$$

If an average value of $Z_x/S_x = 1.1$ is used (this is conservative),

$$F_b = 0.6F_y(1.1) = 0.66F_y$$

If this value is used in Example 5.3,

$$F_b = 0.66(50) = 33.0 \text{ ksi}$$

which is conservative by about 4%. Thus, for compact, laterally-supported beams, the allowable stress can be conservatively taken as $0.66F_y$. (This value of allowable stress has been used in AISC allowable stress design specifications since 1963.)

We can formulate an allowable stress approach that requires no approximation if we use the plastic section modulus instead of the elastic section modulus. From

$$\frac{M_n}{\Omega_b} \geq M_a$$

and

$$\frac{M_n}{\Omega_b} = \frac{F_yZ_y}{1.67} = 0.6F_yZ_x$$

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The required plastic section modulus is

$$Z_x \geq \frac{M_a}{0.6F_y}$$

Thus, if the bending stress is based on the plastic section modulus Z_x,

$$f_b = \frac{M_a}{Z_x} \quad \text{and} \quad F_b = 0.6F_y$$

This approach is useful when designing compact, laterally-supported beams.

The moment strength of compact shapes is a function of the unbraced length, L_b, defined as the distance between points of lateral support, or bracing. In this book, we indicate points of lateral support with an "×," as shown in Figure 5.12. The relationship between the nominal strength, M_n, and the unbraced length is shown in Figure 5.13. If the unbraced length is no greater than L_p, to be defined presently, the beam is considered to have full lateral support, and $M_n = M_p$. If L_b is greater than L_p but less than or equal to the parameter L_r, the strength is based on inelastic LTB. If L_b is greater than L_r, the strength is based on elastic LTB.

FIGURE 5.12

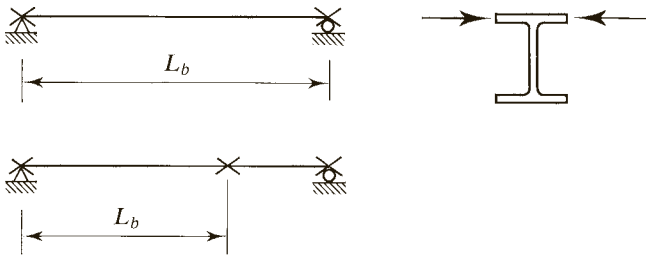
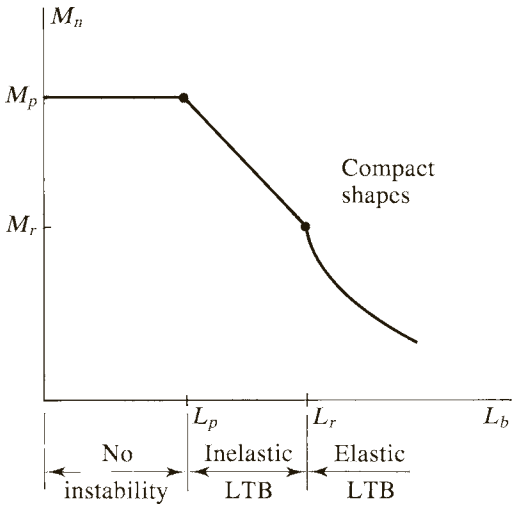


FIGURE 5.13



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The equation for the theoretical elastic lateral-torsional buckling strength can be found in *Theory of Elastic Stability* (Timoshenko and Gere, 1961). With some notational changes, the nominal moment strength is

$$M_n = F_{cr} S_x$$

where F_{cr} is the elastic buckling stress and is given by

$$F_{cr} = \frac{\pi}{L_b S_x} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}, \text{ ksi} \tag{5.5}$$

where

L_b = unbraced length (in.)
I_y = moment of inertia about the weak axis of the cross section (in.^{4})
G = shear modulus of structural steel = 11,200 ksi
J = torsional constant (in.^{4})
C_w = warping constant (in.^{6})

(The constants G, J, and C_w were defined in Chapter 4 in the discussion of torsional and lateral-torsional buckling of columns.)

Equation 5.5 is valid as long as the bending moment within the unbraced length is uniform (nonuniform moment is accounted for with a factor C_b, which is explained later). The AISC Specification gives a different, but equivalent, form for the elastic buckling stress F_{cr}. AISC gives the nominal moment strength as

$$M_n = F_{cr} S_x \leq M_p \qquad \text{(AISC Equation F2-3)}$$

where

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_0} \left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(AISC Equation F2-4)}$$

and

C_b = factor to account for nonuniform bending within the unbraced length L_b. This factor will be covered following Example 5.4.

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \qquad \text{(AISC Equation F2-7)}$$

c = 1.0 for doubly symmetric I shapes (AISC Equation F2-8a)

$$= \frac{h_0}{2} \sqrt{\frac{I_y}{C_w}} \quad \text{for channels} \qquad \text{(AISC Equation F2-8b)}$$

h_0 = distance between flange centroids = $d - t_f$

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If the moment when lateral-torsional buckling occurs is greater than the moment corresponding to first yield, the strength is based on inelastic behavior. The moment corresponding to first yield is

$$M_r = 0.7F_yS_x$$

where the yield stress has been reduced by 30% to account for the effect of residual stress. As shown in Figure 5.13, the boundary between elastic and inelastic behavior will be for an unbraced length of L_r, which is the value of L_b obtained from AISC Equation F2-4 when F_{cr} is set equal to $0.7F_y$ with $C_b = 1.0$. The following equation results:

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_0} + \sqrt{\left(\frac{Jc}{S_xh_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \quad \text{(AISC Equation F2-6)}$$

As with columns, inelastic buckling of beams is more complicated than elastic buckling, and empirical formulas are often used. The following equation is used by AISC:

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(AISC Equation F2-2)}$$

where the $0.7F_yS_x$ term is the yield moment adjusted for residual stress, and

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} \quad \text{(AISC Equation F2-5)}$$

Summary of Nominal Flexural Strength

The nominal bending strength for compact I and C-shaped sections can be summarized as follows:

For $L_b \le L_p$,

$$M_n = M_p \quad \text{(AISC Equation F2-1)}$$

For $L_p < L_b \le L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \quad \text{(AISC Equation F2-2)}$$

For $L_b > L_r$,

$$M_n = F_{cr}S_x \le M_p \quad \text{(AISC Equation F2-3)}$$

where

$$F_{cr} = \frac{C_b\pi^2E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{(AISC Equation F2-4)}$$

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Example 5.4

Determine the flexural strength of a W14 × 68 of A572 Grade 50 steel subject to

a. Continuous lateral support.
b. An unbraced length of 20 ft with $C_b = 1.0$.
c. An unbraced length of 30 ft with $C_b = 1.0$.

Solution To determine the yield stress of a W14 × 68 of A572 Grade 50 steel, we refer to Table 2-4 in Part 2 of the *Manual*. As the designation implies, the yield stress, F_y, is 50 ksi. Next, determine whether this shape is compact, noncompact, or slender:

$$\frac{b_f}{2t_f} = 6.97 \quad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15 > 6.97 \qquad \therefore \text{ The flange is compact.}$$

The web is compact for all shapes in the *Manual* for $F_y \leq 65$ ksi; therefore, a W14 × 68 is compact for $F_y = 50$ ksi. (This determination could also be made by observing that there is no footnote in the dimensions and properties tables to indicate that the shape is not compact.)

a. Because the beam is compact and laterally supported, the nominal flexural strength is

$$M_n = M_p = F_y Z_x = 50(115) = 5750 \text{ in.-kips} = 479.2 \text{ ft-kips}$$

LRFD solution The design strength is

$$\phi_b M_n = 0.90(479.2) = 431 \text{ ft-kips}$$

ASD solution The allowable moment strength is

$$\frac{M_n}{\Omega_b} = \frac{M_n}{1.67} = 0.6M_n = 0.6(479.2) = 288 \text{ ft-kips}$$

b. $L_b = 20$ ft and $C_b = 1.0$. First, determine L_p and L_r:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} = 1.76(2.46)\sqrt{\frac{29{,}000}{50}} = 104.3 \text{ in.} = 8.692 \text{ ft}$$

The following terms will be needed in the computation of L_r:

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} = \frac{\sqrt{121(5380)}}{103} = 7.833 \text{ in.}^2$$

$$r_{ts} = \sqrt{7.833} = 2.799 \text{ in.}$$

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(r_{ts} can also be found in the dimensions and properties tables. For a W14 × 68, it is given as 2.80 in.)

$$h_o = d - t_f = 14.0 - 0.720 = 13.28 \text{ in.}$$

(h_o can also be found in the dimensions and properties tables. For a W14 × 68, it is given as 13.3 in.)

For a doubly symmetric I shape, $c = 1.0$. From AISC Equation F2-6,

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}}$$

$$\frac{Jc}{S_x h_o} = \frac{3.01(1.0)}{103(13.28)} = 0.002201$$

$$L_r = 1.95(2.799)\frac{29{,}000}{0.7(50)} \sqrt{0.002201 + \sqrt{(0.002201)^2 + 6.76\left[\frac{0.7(50)}{29{,}000}\right]^2}}$$

$$= 351.3 \text{ in.} = 29.28 \text{ ft}$$

Since $L_p < L_b < L_r$,

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

$$= 1.0\left[5{,}750 - (5{,}750 - 0.7 \times 50 \times 103)\left(\frac{20 - 8.692}{29.28 - 8.692}\right)\right]$$

$$= 4572 \text{ in.-kips} = 381.0 \text{ ft-kips} < M_p = 479.2 \text{ ft.-kips}$$

LRFD solution The design strength is

$$\phi_b M_n = 0.90(381.0) = 343 \text{ ft-kips}$$

ASD solution The allowable moment strength is

$$\frac{M_n}{\Omega_b} = 0.6M_n = 0.6(381.0) = 229 \text{ ft-kips}$$

c. $L_b = 30$ ft and $C_b = 1.0$. So $L_b > L_r = 29.28$ ft, and elastic lateral-torsional buckling controls.

From AISC Equation F2-4,

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.0\pi^2(29{,}000)}{\left(\frac{30 \times 12}{2.799}\right)^2} \sqrt{1 + 0.078\,\frac{3.01(1.0)}{103(13.28)}\left(\frac{30 \times 12}{2.799}\right)^2} = 33.90 \text{ ksi}$$

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From AISC Equation F2-3,

$$M_n = F_{cr}S_x = 33.90(103) = 3492 \text{ in.-kips} = 291.0 \text{ ft-kips} < M_p = 479.2 \text{ ft-kips}$$

LRFD solution $\phi_b M_n = 0.90(291.0) = 262$ ft-kips

ASD solution $M_n/\Omega_b = 0.6M_n = 0.6(291.0) = 175$ ft-kips

If the moment within the unbraced length L_b is uniform (constant), there is no *moment gradient* and $C_b = 1.0$. If there is a moment gradient, the value of C_b is given by

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad \text{(AISC Equation F1-1)}$$

where

M_{max} = absolute value of the maximum moment within the unbraced length (including the end points of the unbraced length)
M_A = absolute value of the moment at the quarter point of the unbraced length
M_B = absolute value of the moment at the midpoint of the unbraced length
M_C = absolute value of the moment at the three-quarter point of the unbraced length

AISC Equation F1-1 is valid for doubly symmetric members and for singly symmetric members in single curvature.

When the bending moment is uniform, the value of C_b is

$$C_b = \frac{12.5M}{2.5M + 3M + 4M + 3M} = 1.0$$

Example 5.5

Determine C_b for a uniformly loaded, simply supported W shape with lateral support at its ends only.

Solution Because of symmetry, the maximum moment is at midspan, so

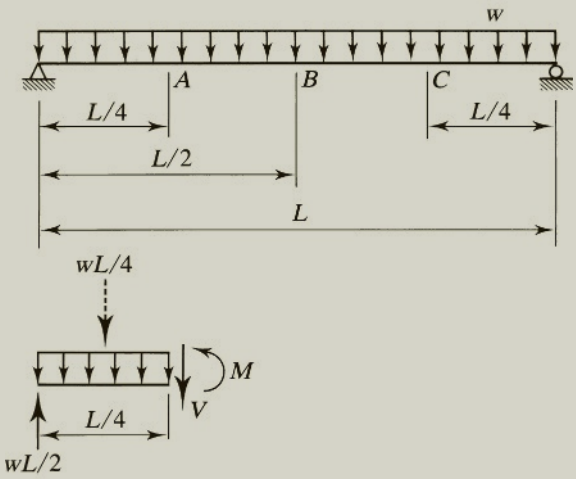
$$M_{max} = M_B = \frac{1}{8}wL^2$$

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Also because of symmetry, the moment at the quarter point equals the moment at the three-quarter point. From Figure 5.14,

$$M_A = M_C = \frac{wL}{2}\left(\frac{L}{4}\right) - \frac{wL}{4}\left(\frac{L}{8}\right) = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{3}{32}wL^2$$

FIGURE 5.14



Since this is a W shape (doubly symmetric), AISC Equation (F1-1) is applicable.

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5\left(\frac{1}{8}\right)}{2.5\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right) + 4\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right)} = 1.14$$

Answer $C_b = 1.14$

Figure 5.15 shows the value of C_b for several common cases of loading and lateral support. Values of C_b for other cases can be found in Part 3 of the *Manual*, "Design of Flexural Members."

For unbraced cantilever beams, AISC specifies a value of C_b of 1.0 (if there is no warping at the support). A value of 1.0 is always conservative, regardless of beam configuration or loading, but in some cases it may be excessively conservative.

The effect of C_b on the nominal strength is illustrated in Figure 5.16. Although the strength is directly proportional to C_b, this graph clearly shows the

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FIGURE 5.15

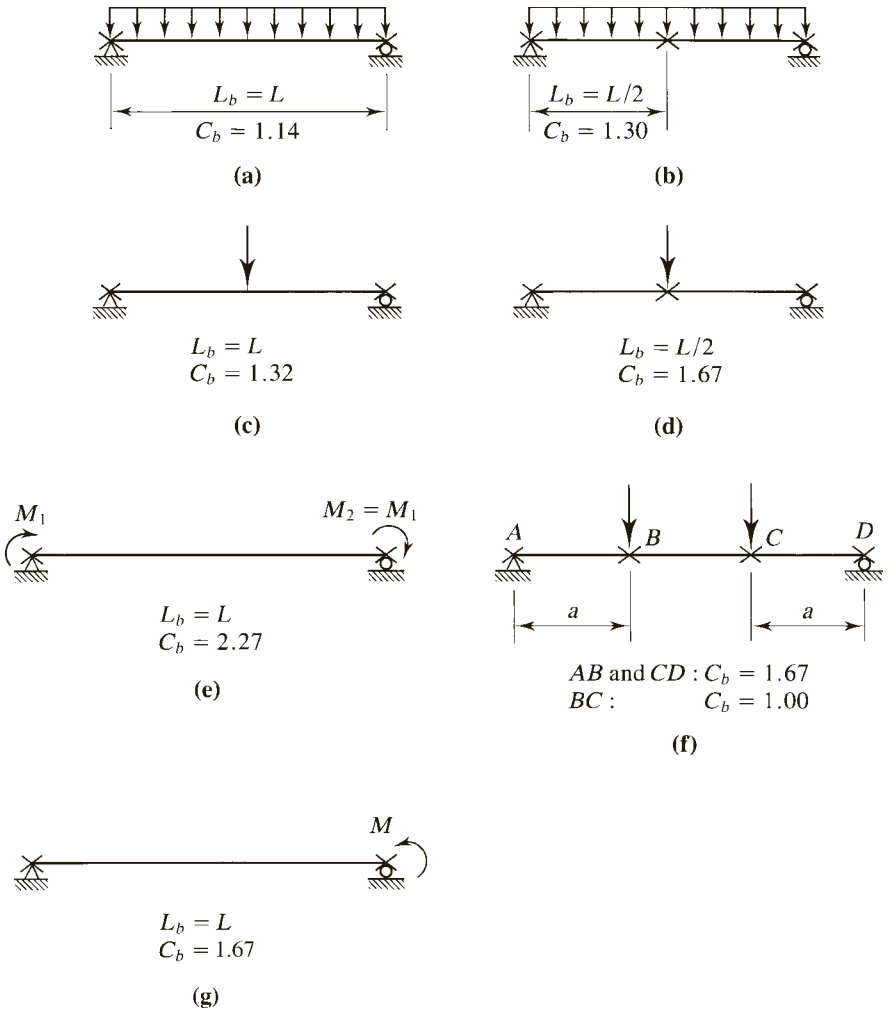
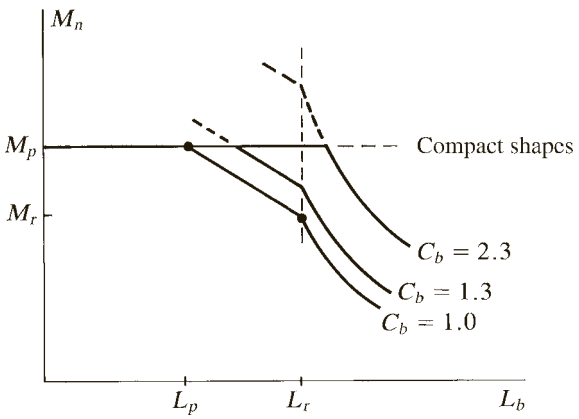


FIGURE 5.16



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importance of observing the upper limit of M_p, regardless of which equation is used for M_n.

Part 3 of the *Steel Construction Manual,* "Design of Flexural Members," contains several useful tables and charts for the analysis and design of beams. For example, Table 3-2, "W Shapes, Selection by Z_x," (hereafter referred to as the "Z_x table"), lists shapes commonly used as beams, arranged in order of available flexural strength—both $\phi_b M_{px}$ and M_{px}/Ω_b. Other useful constants that are tabulated include L_p and L_r (which is particularly tedious to compute). These two constants can also be found in several other tables in Part 3 of the *Manual.* We cover additional design aids in other sections of this chapter.

5.6 BENDING STRENGTH OF NONCOMPACT SHAPES

As previously noted, most standard W, M, S, and C shapes are compact. A few are noncompact because of the flange width-to-thickness ratio, but none are slender.

In general, a noncompact beam may fail by lateral-torsional buckling, flange local buckling, or web local buckling. Any of these types of failure can be in either the elastic range or the inelastic range. The strength corresponding to each of these three limit states must be computed, and the smallest value will control.

From AISC F3, for flange local buckling, if $\lambda_p < \lambda \leq \lambda_r$, the flange is noncompact, buckling will be inelastic, and

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{(AISC Equation F3-1)}$$

where

$$\lambda = \frac{b_f}{2t_f}$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}}$$

The webs of all hot-rolled shapes in the *Manual* are compact, so the noncompact shapes are subject only to the limit states of lateral-torsional buckling and flange local buckling. Built-up welded shapes, however, can have noncompact or slender webs as well as noncompact or slender flanges. These cases are covered in AISC Sections F4 and F5. Built-up shapes, including plate girders, are covered in Chapter 10 of this textbook.

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Example 5.6

A simply supported beam with a span length of 45 feet is laterally supported at its ends and is subjected to the following service loads:

Dead load = 400 lb/ft (including the weight of the beam)

Live load = 1000 lb/ft

If $F_y = 50$ ksi, is a W14 × 90 adequate?

Solution Determine whether the shape is compact, noncompact, or slender:

$$\lambda = \frac{b_f}{2t_f} = 10.2$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000}{50}} = 9.15$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29{,}000}{50}} = 24.1$$

Since $\lambda_p < \lambda < \lambda_r$, this shape is noncompact. Check the capacity based on the limit state of flange local buckling:

$$M_p = F_y Z_x = 50(157) = 7850 \text{ in.-kips}$$

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

$$= 7850 - (7850 - 0.7 \times 50 \times 143)\left(\frac{10.2 - 9.15}{24.1 - 9.15}\right)$$

$$= 7650 \text{ in.-kips} = 637.5 \text{ ft-kips}$$

Check the capacity based on the limit state of lateral-torsional buckling. From the Z_x table,

$L_p = 15.1$ ft and $L_r = 42.5$ ft

$L_b = 45 \text{ ft} > L_r$ $\quad \therefore$ Failure is by *elastic* LTB.

From Part 1 of the *Manual,*

$I_y = 362 \text{ in.}^4$
$r_{ts} = 4.11$ in.
$h_o = 13.3$ in.
$J = 4.06 \text{ in.}^4$
$C_w = 16{,}000 \text{ in.}^6$

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For a uniformly loaded, simply supported beam with lateral support at the ends,

$C_b = 1.14$ (Fig. 5.15a)

For a doubly symmetric I shape, $c = 1.0$. AISC Equation F2-4 gives

$$F_{cr} = \frac{C_b \pi^2 E}{(L_b/r_{ts})^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= \frac{1.14\pi^2(29{,}000)}{\left(\frac{45 \times 12}{4.11}\right)^2} \sqrt{1 + 0.078 \frac{4.06(1.0)}{143(13.3)} \left(\frac{45 \times 12}{4.11}\right)^2} = 37.20 \text{ ksi}$$

From AISC Equation F2-3,

$$M_n = F_{cr} S_x = 37.20(143) = 5320 \text{ in.-kips} < M_p = 7850 \text{ in.-kips}$$

This is smaller than the nominal strength based on flange local buckling, so lateral-torsional buckling controls.

LRFD solution The design strength is

$$\phi_b M_n = 0.90(5320) = 4788 \text{ in.-kips} = 399 \text{ ft-kips}$$

The factored load and moment are

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.400) + 1.6(1.000) = 2.080 \text{ kips/ft}$$

$$M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(2.080)(45)^2 = 527 \text{ ft-kips} > 399 \text{ ft-kips} \qquad \text{(N.G.)}$$

Answer Since $M_u > \phi_b M_n$, the beam does not have adequate moment strength.

ASD solution The allowable stress is

$$F_b = 0.6F_{cr} = 0.6(37.20) = 22.3 \text{ ksi}$$

The applied bending moment is

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(0.400 + 1.000)(45)^2 = 354.4 \text{ ft-kips}$$

and the applied stress is

$$f_b = \frac{M_a}{S_x} = \frac{354.4(12)}{143} = 29.7 \text{ ksi} > 22.3 \text{ ksi} \qquad \text{(N.G.)}$$

Answer Since $f_b > F_b$, the beam does not have adequate moment strength.

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Noncompact shapes are identified in the Z_x table by an "*f*" footnote (this same identification is used in the dimensions and properties tables). Noncompact shapes are also treated differently in the Z_x table in the following way. The tabulated value of L_p is the value of unbraced length at which the nominal strength based on inelastic lateral-torsional buckling equals the nominal strength based on flange local buckling, that is, the maximum unbraced length for which the nominal strength can be taken as the strength based on flange local buckling. (Recall that L_p for compact shapes is the maximum unbraced length for which the nominal strength can be taken as the plastic moment.) For the shape in Example 5.6, equate the nominal strength based on FLB to the strength based on inelastic LTB (AISC Equation F2-2), with $C_b = 1.0$:

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right) \tag{5.6}$$

The value of L_r was given in Example 5.6 and is unchanged. The value of L_p, however, must be computed from AISC Equation F2-5:

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} = 1.76(3.70)\sqrt{\frac{29{,}000}{50}} = 156.8 \text{ in.} = 13.07 \text{ ft}$$

Returning to Equation 5.6, we obtain

$$7650 = 7850 - (7850 - 0.7 \times 50 \times 143)\left(\frac{L_b - 13.07}{42.6 - 13.07}\right)$$

$$L_b = 15.2 \text{ ft}$$

This is the value tabulated as L_p for a W14 × 90 with $F_y = 50$ ksi. Note that

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}}$$

could still be used for noncompact shapes. If doing so resulted in the equation for inelastic LTB being used when L_b was not really large enough, the strength based on FLB would control anyway.

In addition to the different meaning of L_p for noncompact shapes in the Z_x table, the available strength values, $\phi_b M_{px}$ and M_p/Ω_b, are based on flange local buckling rather than the plastic moment.

5.7 SUMMARY OF MOMENT STRENGTH

The procedure for computation of nominal moment strength for I and C-shaped sections bent about the *x* axis will now be summarized. All terms in the following equations have been previously defined, and AISC equation numbers will not be shown.

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This summary is for compact and noncompact shapes (noncompact flanges) only (no slender shapes).

1. Determine whether the shape is compact.
2. If the shape is compact, check for lateral-torsional buckling as follows.

   If $L_b \le L_p$, there is no LTB, and $M_n = M_p$

   If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

   If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{\text{cr}}S_x \le M_p$$

   where

$$F_{\text{cr}} = \frac{C_b\pi^2E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

3. If the shape is noncompact because of the flange, the nominal strength will be the smaller of the strengths corresponding to flange local buckling and lateral-torsional buckling.

   a. Flange local buckling:

   If $\lambda \le \lambda_p$, there is no FLB

   If $\lambda_p < \lambda \le \lambda_r$, the flange is noncompact, and

$$M_n = M_p - (M_p - 0.7F_yS_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)$$

   b. Lateral-torsional buckling:

   If $L_b \le L_p$, there is no LTB

   If $L_p < L_b \le L_r$, there is inelastic LTB, and

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

   If $L_b > L_r$, there is elastic LTB, and

$$M_n = F_{\text{cr}}S_x \le M_p$$

   where

$$F_{\text{cr}} = \frac{C_b\pi^2E}{(L_b/r_{ts})^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_0}\left(\frac{L_b}{r_{ts}}\right)^2}$$

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5.8 SHEAR STRENGTH

Beam shear strength is covered in Chapter G of the AISC Specification, "Design of Members for Shear." Both hot-rolled shapes and welded built-up shapes are covered. We discuss hot-rolled shapes in the present chapter of this book and built-up shapes in Chapter 10, "Plate Girders." The AISC provisions for hot-rolled shapes are covered in Section G2.1.

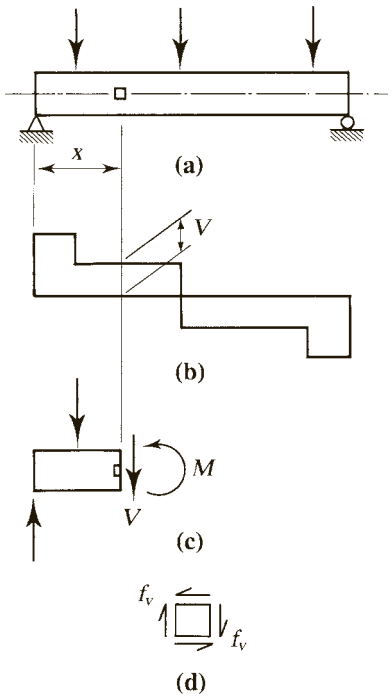
Before covering the AISC provisions for shear strength, we will first review some basic concepts from mechanics of materials. Consider the simple beam of Figure 5.17. At a distance x from the left end and at the neutral axis of the cross section, the state of stress is as shown in Figure 5.17d. Because this element is located at the neutral axis, it is not subjected to flexural stress. From elementary mechanics of materials, the shearing stress is

$$f_v = \frac{VQ}{Ib} \tag{5.7}$$

where

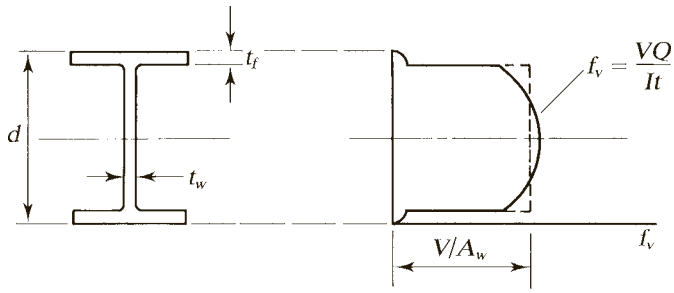
f_v = vertical and horizontal shearing stress at the point of interest
V = vertical shear force at the section under consideration
Q = first moment, about the neutral axis, of the area of the cross section between the point of interest and the top or bottom of the cross section
I = moment of inertia about the neutral axis
b = width of the cross section at the point of interest

FIGURE 5.17



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FIGURE 5.18



Equation 5.7 is based on the assumption that the stress is constant across the width b, and it is therefore accurate only for small values of b. For a rectangular cross section of depth d and width b, the error for $d/b = 2$ is approximately 3%. For $d/b = 1$, the error is 12% and for $d/b = 1/4$, it is 100% (Higdon, Ohlsen, and Stiles, 1960). For this reason, Equation 5.7 cannot be applied to the flange of a W-shape in the same manner as for the web.

Figure 5.18 shows the shearing stress distribution for a W shape. Superimposed on the actual distribution is the average stress in the web, V/A_w, which does not differ much from the maximum web stress. Clearly, the web will completely yield long before the flanges begin to yield. Because of this, yielding of the web represents one of the shear limit states. Taking the shear yield stress as 60% of the tensile yield stress, we can write the equation for the stress in the web at failure as

$$f_v = \frac{V_n}{A_w} = 0.6F_y$$

where A_w = area of the web. The nominal strength corresponding to this limit state is therefore

$$V_n = 0.6F_yA_w \tag{5.8}$$

and will be the nominal strength in shear provided that there is no shear buckling of the web. Whether that occurs will depend on h/t_w, the width-to-thickness ratio of the web. If this ratio is too large—that is, if the web is too slender—the web can buckle in shear, either inelastically or elastically.

AISC Specification Requirements for Shear

For LRFD, the relationship between required and available strength is

$$V_u \leq \phi_v V_n$$

where

V_u = maximum shear based on the controlling combination of factored loads
ϕ_v = resistance factor for shear

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For ASD, the relationship is

$$V_a \leq \frac{V_n}{\Omega_v}$$

where

V_a = maximum shear based on the controlling combination of service loads
Ω_v = safety factor for shear

As we will see, the values of the resistance factor and safety factor will depend on the web width-to-thickness ratio.

Section G2.1 of the AISC Specification covers both beams with stiffened webs and beams with unstiffened webs. In most cases, hot-rolled beams will not have stiffeners, and we will defer treatment of stiffened webs until Chapter 10. The basic strength equation is

$$V_n = 0.6F_yA_wC_{v1} \qquad \text{(AISC Equation G2-1)}$$

where

A_w = area of the web $\approx dt_w$
d = overall depth of the beam
C_{v1} = ratio of critical web stress to shear yield stress*

The value of C_{v1} depends on whether the limit state is web yielding or web buckling.

Case 1: For hot-rolled I shapes with

$$\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$$

The limit state is shear yielding, and

$C_{v1} = 1.0$ (AISC Equation G2-2)
$\phi_v = 1.00$
$\Omega_v = 1.50$

Most W shapes with $F_y \leq 50$ ksi fall into this category (see User Note in AISC G2.1[a]).

Case 2: For all other I shapes and channels,

$\phi_v = 0.90$
$\Omega_v = 1.67$

*Since there is a C_{v1}, there is obviously a C_{v2}. The constant C_{v2} is used in the shear equations for plate girders in Chapter 10 of this book.

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and C_{v1} is determined as follows:

For $\dfrac{h}{t_w} \le 1.10\sqrt{\dfrac{k_v E}{F_y}}$, there is no web instability, and

$$C_{v1} = 1.0 \qquad \text{(AISC Equation G2-3)}$$

(This corresponds to Equation 5.8 for shear yielding.)

For $\dfrac{h}{t_w} > 1.10\sqrt{\dfrac{k_v E}{F_y}}$,

$$C_{v1} = \frac{1.10\sqrt{\dfrac{k_v E}{F_y}}}{h/t_w} \qquad \text{(AISC Equation G2-4)}$$

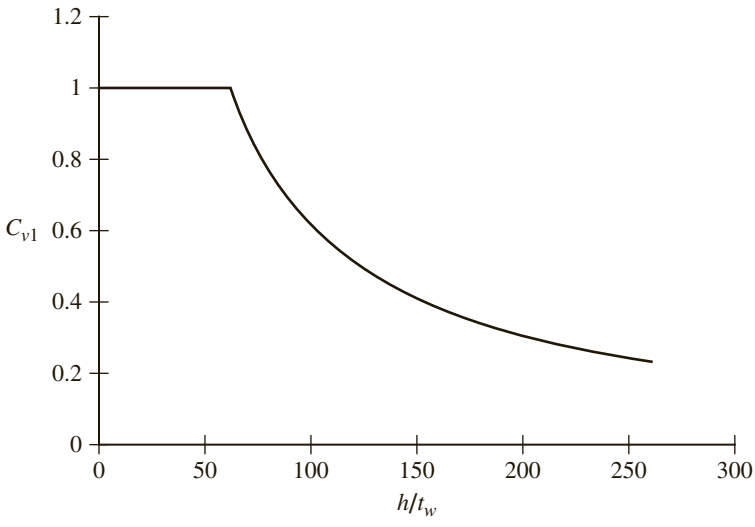
where

$$k_v = 5.34$$

This value of k_v is for unstiffened webs. Webs with stiffeners are covered in Chapter 10, "Plate Girders."

AISC Equation G2-3 is based on the limit state of web shear yielding. AISC Equation G2-4 accounts for the web buckling and post-buckling strengths. The web buckling strength is the shear that brings the web to the point of buckling, and the post-buckling strength is the extra strength available after web buckling has occurred. The relationship between web width-to-thickness ratio and strength (which is a function of C_{v1}) is shown in Figure 5.19. The flat portion corresponds to web shear yielding, and the curved part corresponds to buckling and post-buckling strength.

FIGURE 5.19



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Allowable Stress Formulation

The allowable strength relation

$$V_a \le \frac{V_n}{\Omega_v}$$

can also be written in terms of stress as

$$f_v \le F_v$$

where

$$f_v = \frac{V_a}{A_w} = \text{ applied shear stress}$$

$$F_v = \frac{V_n/\Omega_v}{A_w} = \frac{0.6F_yA_wC_{v1}/\Omega_v}{A_w} = \text{allowable shear stress}$$

For the most common case of hot-rolled I shapes with $h/t_w \le 2.24\sqrt{E/F_y}$,

$$F_v = \frac{0.6F_yA_w(1.0)/1.50}{A_w} = 0.4F_y$$

Shear is rarely a problem in rolled steel beams; the usual practice is to design a beam for flexure and then to check it for shear.

Example 5.7

Check the beam in Example 5.6 for shear.

Solution From the dimensions and properties tables in Part 1 of the *Manual,* the web width-to-thickness ratio of a W14 × 90 is

$$\frac{h}{t_w} = 25.9$$

and the web area is $A_w = dt_w = 14.0(0.440) = 6.160 \text{ in.}^2$

$$2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{29{,}000}{50}} = 54.0$$

Since

$$\frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}}$$

the strength is governed by shear yielding of the web and $C_{v1} = 1.0$. (As pointed out in the Specification User Note, this will be the case for most W shapes with $F_y \le 50$ ksi.) The nominal shear strength is

$$V_n = 0.6F_yA_wC_{v1} = 0.6(50)(6.160)(1.0) = 184.8 \text{ kips}$$

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LRFD solution Determine the resistance factor ϕ_v.

$$\text{Since } \frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}},$$

$$\phi_v = 1.00$$

and the design shear strength is

$$\phi_v V_n = 1.00(184.8) = 185 \text{ kips}$$

From Example 5.6, $w_u = 2.080$ kips/ft and $L = 45$ ft. For a simply supported, uniformly loaded beam, the maximum shear occurs at the support and is equal to the reaction.

$$V_u = \frac{w_u L}{2} = \frac{2.080(45)}{2} = 46.8 \text{ kips} < 185 \text{ kips} \qquad \text{(OK)}$$

ASD solution Determine the safety factor Ω_v.

$$\text{Since } \frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}},$$

$$\Omega_v = 1.50$$

and the allowable shear strength is

$$\frac{V_n}{\Omega_v} = \frac{184.8}{1.50} = 123 \text{ kips}$$

From Example 5.6, the total service load is

$$w_a = w_D + w_L = 0.400 + 1.000 = 1.4 \text{ kips/ft}$$

The maximum shear is

$$V_a = \frac{w_a L}{2} = \frac{1.4(45)}{2} = 31.5 \text{ kips} < 123 \text{ kips} \qquad \text{(OK)}$$

Alternately, a solution in terms of stress can be done. Since shear yielding controls ($C_{v1} = 1.0$) and $\Omega_v = 1.50$, the allowable shear stress is

$$F_v = 0.4F_y = 0.4(50) = 20 \text{ ksi}$$

The required shear strength (stress) is

$$f_a = \frac{V_a}{A_w} = \frac{31.5}{6.160} = 5.11 \text{ ksi} < 20 \text{ ksi} \qquad \text{(OK)}$$

Answer The required shear strength is less than the available shear strength, so the beam is satisfactory.

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Values of $\phi_v V_n$ and V_n/Ω_v are given in several tables in Part 3 of the *Manual,* including the Z_x table, so computation of shear strength is unnecessary for hot-rolled shapes.

Block Shear

Block shear, which was considered earlier in conjunction with tension member connections, can occur in certain types of beam connections. To facilitate the connection of beams to other beams so that the top flanges are at the same elevation, a short length of the top flange of one of the beams may be cut away, or *coped*. If a coped beam is connected with bolts as in Figure 5.20, segment ABC will tend to tear out. The applied load in this case will be the vertical beam reaction, so shear will occur along line AB and there will be tension along BC. Thus the block shear strength will be a limiting value of the reaction.

We covered the computation of block shear strength in Chapter 3, but we will review it here. Failure is assumed to occur by rupture (fracture) on the shear area (subject to an upper limit) and rupture on the tension area. AISC J4.3, "Block Shear Strength," gives the following equation for block shear strength:

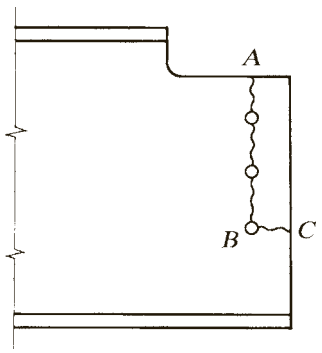
$$R_n = 0.6F_u A_{nv} + U_{bs}F_u A_{nt} \leq 0.6F_y A_{gv} + U_{bs}F_u A_{nt} \qquad \text{(AISC Equation J4-5)}$$

where

A_{gv} = gross area in shear (in Figure 5.20, length AB times the web thickness)
A_{nv} = net area along the shear surface or surfaces
A_{nt} = net area along the tension surface (in Figure 5.20, along BC)
U_{bs} = 1.0 when the tensile stress is uniform (for most coped beams)
= 0.5 when the tension stress is not uniform (coped beams with two lines of bolts or with nonstandard distance from bolts to end of beam) (Ricles and Yura, 1983)

For LRFD, $\phi = 0.75$. For ASD, $\Omega = 2.00$.

FIGURE 5.20

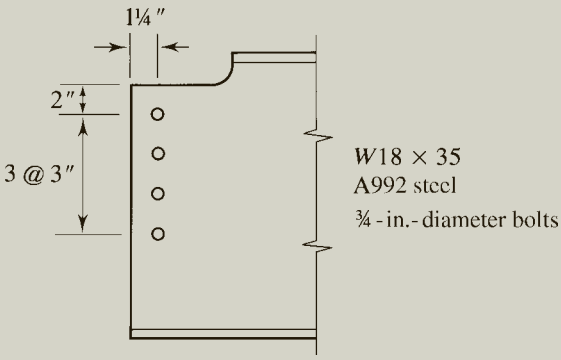


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Example 5.8

Determine the maximum reaction, based on block shear, that can be resisted by the beam shown in Figure 5.21.

FIGURE 5.21



Solution The effective hole diameter is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in. The shear areas are

$$A_{gv} = t_w(2 + 3 + 3 + 3) = 0.300(11) = 3.300 \text{ in.}^2$$

$$A_{nv} = 0.300\left[11 - 3.5\left(\frac{7}{8}\right)\right] = 2.381 \text{ in.}^2$$

The net tension area is

$$A_{nt} = 0.300\left[1.25 - \frac{1}{2}\left(\frac{7}{8}\right)\right] = 0.2438 \text{ in.}^2$$

Since the block shear will occur in a coped beam with a single line of bolts, $U_{bs} = 1.0$. From AISC Equation J4-5,

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} = 0.6(65)(2.381) + 1.0(65)(0.2438) = 108.7 \text{ kips}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(65)(3.300) + 1.0(65)(0.2438) = 144.5 \text{ kips}$$

The nominal block shear strength is therefore 108.7 kips.

LRFD solution The maximum factored load reaction is the design strength: $\phi R_n = 0.75(108.7) = 81.5$ kips.

Answer Design strength = 81.5 kips

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ASD solution The maximum service load reaction is the allowable strength: $\frac{R_n}{\Omega} = \frac{108.7}{2.00} = 54.4$ kips.

Answer Allowable strength = 54.4 kips

5.9 DEFLECTION

In addition to being safe, a structure must be *serviceable*. A serviceable structure is one that performs satisfactorily, not causing any discomfort or perceptions of unsafety for the occupants or users of the structure. For a beam, being serviceable usually means that the deformations, primarily the vertical sag, or deflection, must be limited. Excessive deflection is usually an indication of a very flexible beam, which can lead to problems with vibrations. The deflection itself can cause problems if elements attached to the beam can be damaged by small distortions. In addition, users of the structure may view large deflections negatively and wrongly assume that the structure is unsafe.

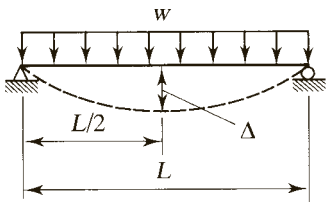
For the common case of a simply supported, uniformly loaded beam such as that in Figure 5.22, the maximum vertical deflection is

$$\Delta = \frac{5}{384}\frac{wL^4}{EI}$$

Deflection formulas for a variety of beams and loading conditions can be found in Part 3, "Design of Flexural Members," of the *Manual*. For more unusual situations, standard analytical methods such as the method of virtual work may be used. Deflection is a serviceability limit state, not one of strength, so deflections should always be computed with *service* loads.

The appropriate limit for the maximum deflection depends on the function of the beam and the likelihood of damage resulting from the deflection. The AISC Specification furnishes little guidance other than a statement in Chapter L, "Design for Serviceability," that deflections should not be excessive. There is, however, a more detailed discussion in the Commentary to Chapter L. Appropriate limits for deflection can usually be found from the governing building code, expressed as a fraction of the span length L, such as $L/360$. Sometimes a numerical limit, such as 1 inch, is appropriate. The limits given in the International

FIGURE 5.22



$$\Delta = \frac{5}{384}\frac{wL^4}{EI}$$

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TABLE 5.4 Deflection Limits

Type of member	Max. live load defl.	Max. dead + live load defl.	Max. snow or wind load defl.
Roof beam:			
Supporting plaster ceiling	$L/360$	$L/240$	$L/360$
Supporting nonplaster ceiling	$L/240$	$L/180$	$L/240$
Not supporting a ceiling	$L/180$	$L/120$	$L/180$
Floor beam	$L/360$	$L/240$	—

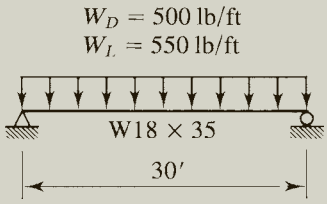
Building Code (ICC, 2015) are typical. Table 5.4 shows some of the deflection limits given by that code.

The limits shown in Table 5.4 for deflection due to dead load plus live load do not apply to steel beams, because the dead load deflection is usually compensated for by some means, such as *cambering*. Camber is a curvature in the opposite direction of the dead load deflection curve and can be accomplished by bending the beam, with or without heat. When the dead load is applied to the cambered beam, the curvature is removed, and the beam becomes level. Therefore, only the live load deflection is of concern in the completed structure. Dead load deflection can also be accounted for by pouring a variable depth slab with a level top surface, the variable depth being a consequence of the deflection of the beam (this is referred to as *ponding* of the concrete). Detailed coverage of control of dead load deflection is given in an AISC seminar series (AISC, 1997a) and several papers (Ruddy, 1986; Ricker, 1989; and Larson and Huzzard, 1990).

Example 5.9

Compute the dead load and live load deflections for the beam shown in Figure 5.23. If the maximum permissible *live* load deflection is $L/360$, is the beam satisfactory?

FIGURE 5.23



Solution It is more convenient to express the deflection in inches than in feet, so units of inches are used in the deflection formula. The dead load deflection is

$$\Delta_D = \frac{5}{384}\frac{w_D L^4}{EI} = \frac{5}{384}\frac{(0.500/12)(30 \times 12)^4}{29{,}000(510)} = 0.616 \text{ in.}$$

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The live load deflection is

$$\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI} = \frac{5}{384}\frac{(0.550/12)(30 \times 12)^4}{29{,}000(510)} = 0.678 \text{ in.}$$

The maximum permissible live load deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1.0 \text{ in.} > 0.678 \text{ in.} \qquad \text{(OK)}$$

Answer The beam satisfies the deflection criterion.

Ponding is one deflection problem that does affect the safety of a structure. It is a potential hazard for flat roof systems that can trap rainwater. If drains become clogged during a storm, the weight of the water will cause the roof to deflect, thus providing a reservoir for still more water. If this process proceeds unabated, collapse can occur. The AISC specification requires that the roof system have sufficient stiffness to prevent ponding, and it prescribes limits on stiffness parameters in Appendix 2, "Design for Ponding."

5.10 DESIGN

Beam design entails the selection of a cross-sectional shape that will have enough strength and that will meet serviceability requirements. As far as strength is concerned, flexure is almost always more critical than shear, so the usual practice is to design for flexure and then check shear. The design process can be outlined as follows.

1. Compute the required moment strength (i.e., the factored load moment M_u for LRFD or the unfactored moment M_a for ASD). The weight of the beam is part of the dead load but is unknown at this point. A value may be assumed and verified after a shape is selected, or the weight may be ignored initially and checked after a shape has been selected. Because the beam weight is usually a small part of the total load, if it is ignored at the beginning of a design problem, the selected shape will usually be satisfactory when the moment is recomputed. We use the latter approach in this book.
2. Compute the moment of inertia required to meet the deflection limit. Use the service load for this computation.
3. Select a shape that meets the strength requirement, then check the moment of inertia. Select a different shape if necessary.
4. Compute the required moment strength and moment of inertia using the actual beam weight. If necessary, select a different shape.
5. Check the shear strength.

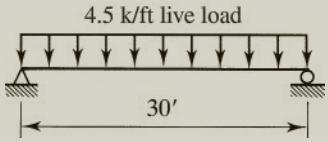
The trial shape can be easily selected in only a limited number of situations (as in Example 5.10). In most cases, design aids, such as the beam design charts in Part 3 of the *Manual*, can be used.

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Example 5.10

Select a standard hot-rolled shape of A992 steel for the beam shown in Figure 5.24. The beam has continuous lateral support and must support a uniform service live load of 4.5 kips/ft. The maximum permissible live load deflection is $L/240$.

FIGURE 5.24



LRFD solution

Ignore the beam weight initially then check for its effect after a selection is made.

$$w_u = 1.2w_D + 1.6w_L = 1.2(0) + 1.6(4.5) = 7.2 \text{ kips/ft}$$

$$\text{Required moment strength } M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(7.2)(30)^2 = 810.0 \text{ ft-kips}$$

$$= \text{required } \phi_b M_n$$

The maximum permissible live load deflection is

$$\frac{L}{240} = \frac{30(12)}{240} = 1.5 \text{ in.}$$

From $\Delta \leq \frac{5}{384}\frac{wL^4}{EI}$, the required moment of inertia is

$$I \geq \frac{5}{384}\frac{wL^4}{E\Delta} = \frac{5}{384}\frac{(4.5/12)(30 \times 12)^4}{29{,}000(1.5)} = 1890 \text{ in.}^4$$

Assume that the shape will be compact. For a compact shape with full lateral support,

$$M_n = M_p = F_y Z_x$$

From $\phi_b M_n \geq M_u$,

$$\phi_b F_y Z_x \geq M_u$$

$$Z_x \geq \frac{M_u}{\phi_b F_y} = \frac{810.0(12)}{0.90(50)} = 216 \text{ in.}^3$$

The Z_x table lists hot-rolled shapes normally used as beams in order of decreasing plastic section modulus. Furthermore, they are grouped so that the shape at the top

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of each group (in bold type) is the lightest one that has enough section modulus to satisfy a required section modulus that falls within the group. In this example, the shape that comes closest to meeting the section modulus requirement is a W21 × 93, with $Z_x = 221$ in.^{3}, but the lightest one is a W24 × 84, with $Z_x = 224$ in.^{3}. Because section modulus is not directly proportional to area, it is possible to have more section modulus with less area, and hence less weight.

Try a W24 × 84: This shape is compact, as assumed (noncompact shapes are marked as such in the table); therefore $M_n = M_p$, as assumed. From the Dimensions and Properties Table, the moment of inertia about the axis of bending is

$$I_x = 2370 \text{ in.}^4 > 1890 \text{ in.}^4 \qquad \text{(OK)}$$

Account for the beam weight:

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.084) + 1.6(4.5) = 7.301 \text{ kips/ft}$$

$$\text{Required moment strength} = M_u = \frac{1}{8} w_u L^2 = \frac{1}{8}(7.301)(30)^2 = 821.4 \text{ ft-kips}$$

The required section modulus is

$$Z_x = \frac{M_u}{\phi_b F_y} = \frac{821.4(12)}{0.90(50)} = 219 \text{ in.}^3 < 224 \text{ in.}^3 \qquad \text{(OK)}$$

In lieu of basing the search on the required section modulus, the design strength $\phi_b M_p$ could be used, because it is directly proportional to Z_x and is also tabulated. The required moment of inertia is unchanged, because the deflection is for live loads, which are unchanged. Next, check the shear.

$$V_u = \frac{w_u L}{2} = \frac{7.301(30)}{2} = 110 \text{ kips}$$

From the Z_x table,

$$\phi_v V_n = 340 \text{ kips} > 110 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W24 × 84.

ASD solution Ignore the beam weight initially, then check for its effect after a selection is made.

$$w_a = w_D + w_L = 0 + 4.5 = 4.5 \text{ kips/ft}$$

$$\text{Required moment strength} = M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(4.5)(30)^2 = 506.3 \text{ ft-kips}$$

$$= \text{required } \frac{M_n}{\Omega_b}$$

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The maximum permissible live load deflection is

$$\frac{L}{240} = \frac{30(12)}{240} = 1.5 \text{ in.}$$

From $\Delta \leq \frac{5}{384}\frac{wL^4}{EI}$, the required moment of inertia is

$$I \geq \frac{5}{384}\frac{wL^4}{E\Delta} = \frac{5}{384}\frac{(4.5/12)(30 \times 12)^4}{29{,}000(1.5)} = 1890 \text{ in.}^4$$

Assume that the shape will be compact. For a compact shape with full lateral support,

$$M_n = M_p = F_y Z_x$$

From $\frac{M_p}{\Omega_b} \geq M_a$,

$$\frac{F_y Z_x}{\Omega_b} \geq M_a$$

$$Z_x \geq \frac{\Omega_b M_a}{F_y} = \frac{1.67(506.3 \times 12)}{50} = 203 \text{ in.}^3$$

The Z_x table lists hot-rolled shapes normally used as beams in order of decreasing plastic section modulus. They are arranged in groups, with the lightest shape in each group at the top of that group. For the current case, the shape with a section modulus closest to 203 in.^{3} is a W18 × 97, but the lightest shape with sufficient section modulus is a W24 × 84, with $Z_x = 224$ in.^{3}

Try a W24 × 84: This shape is compact, as assumed (if it were noncompact, there would be a footnote in the Z_x table). Therefore, $M_n = M_p$ as assumed. From the Dimensions and Properties Table, the moment of inertia about the axis of bending is

$$I_x = 2370 \text{ in.}^4 > 1890 \text{ in.}^4 \qquad \text{(OK)}$$

Account for the beam weight:

$$w_a = w_D + w_L = 0.084 + 4.5 = 4.584 \text{ kips/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(4.584)(30)^2 = 515.7 \text{ ft-kips}$$

The required plastic section modulus is

$$Z_x = \frac{\Omega_b M_a}{F_y} = \frac{1.67(515.7 \times 12)}{50} = 207 \text{ in.}^3 < 224 \text{ in.}^3 \qquad \text{(OK)}$$

The required moment of inertia is unchanged, because the deflection is for live loads, which are unchanged. Instead of searching for the required section modulus,

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the search could be based on the required value of M_p/Ω_b, which is also tabulated. Because M_p/Ω_b is proportional to Z_x, the results will be the same.

Another approach is to use the allowable stress for compact laterally supported shapes. From Section 5.5 of this book, with the flexural stress based on the plastic section modulus,

$$F_b = 0.6F_y = 0.6(50) = 30.0 \text{ ksi}$$

and the required section modulus (before the beam weight is included) is

$$Z_x = \frac{M_a}{F_b} = \frac{506.3 \times 12}{30} = 203 \text{ in.}^3$$

Next, check the shear. The required shear strength is

$$V_a = \frac{w_a L}{2} = \frac{4.584(30)}{2} = 68.8 \text{ kips}$$

From the Z_x table, the available shear strength is

$$\frac{V_n}{\Omega_v} = 227 \text{ kips} > 68.8 \text{ kips} \quad \text{(OK)}$$

Answer Use a W24 × 84.

In Example 5.10, it was first assumed that a compact shape would be used, and then the assumption was verified. However, if the search is made based on available strength ($\phi_b M_p$ or M_p/Ω_b) rather than section modulus, it is irrelevant whether the shape is compact or noncompact. This is because for noncompact shapes, the tablulated values of $\phi_b M_p$ and M_p/Ω_b are based on flange local buckling and not the plastic moment (see Section 5.6). This means that for laterally supported beams, the Z_x table can be used for design without regard to whether the shape is compact or noncompact.

Beam Design Charts

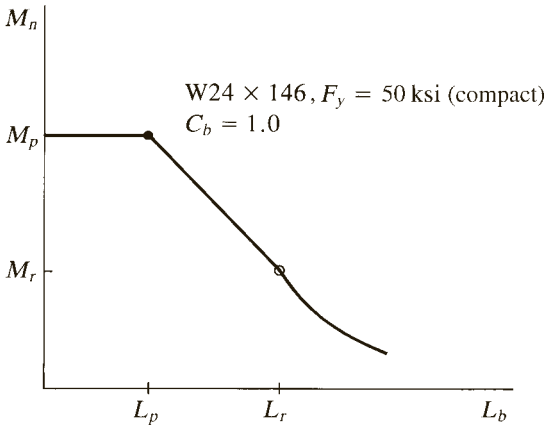
Many graphs, charts, and tables are available for the practicing engineer, and these aids can greatly simplify the design process. For the sake of efficiency, they are widely used in design offices, but you should approach their use with caution and not allow basic principles to become obscured. It is not our purpose to describe in this book all available design aids in detail, but some are worthy of note, particularly the curves of moment strength versus unbraced length given in Part 3 of the *Manual*.

These curves will be described with reference to Figure 5.25, which shows a graph of nominal moment strength as a function of unbraced length L_b for a particular compact shape. Such a graph can be constructed for any cross-sectional shape and specific values of F_y and C_b by using the appropriate equations for moment strength.

The design charts in the *Manual* comprise a family of graphs similar to the one shown in Figure 5.25. Two sets of curves are available, one for W shapes with

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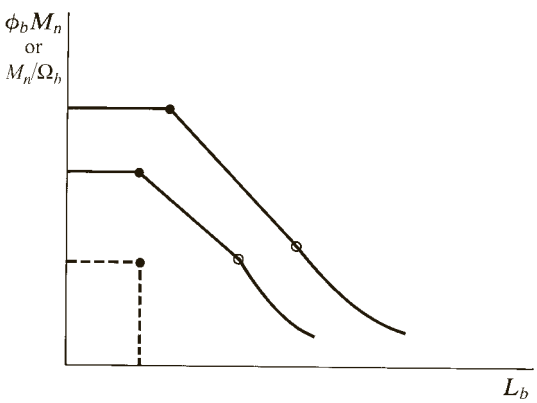
FIGURE 5.25



$F_y = 50$ ksi and one for C and MC shapes with $F_y = 36$ ksi. Each graph gives the flexural strength of a standard hot-rolled shape. Instead of giving the nominal strength M_n, however, both the allowable moment strength M_n/Ω_b and the design moment strength $\phi_b M_n$ are given. Two scales are shown on the vertical axis—one for M_n/Ω_b and one for $\phi_b M_n$. All curves were generated with $C_b = 1.0$. For other values of C_b, simply multiply the moment from the chart by C_b. However, the strength can never exceed the value represented by the horizontal line at the left side of the graph. For a compact shape, this represents the strength corresponding to yielding (reaching the plastic moment M_p). If the curve is for a noncompact shape, the horizontal line represents the flange local buckling strength.

Use of the charts is illustrated in Figure 5.26, where two such curves are shown. Any point on this graph, such as the intersection of the two dashed lines, represents an available moment strength and an unbraced length. If the moment is a required moment capacity, then any curve above the point corresponds to a beam with a larger moment capacity. Any curve to the right is for a beam with exactly the required moment capacity, although for a larger unbraced length. In a design problem, therefore, if the charts are entered with a given unbraced length and a required strength, curves above and to the right of the point correspond to acceptable beams.

FIGURE 5.26



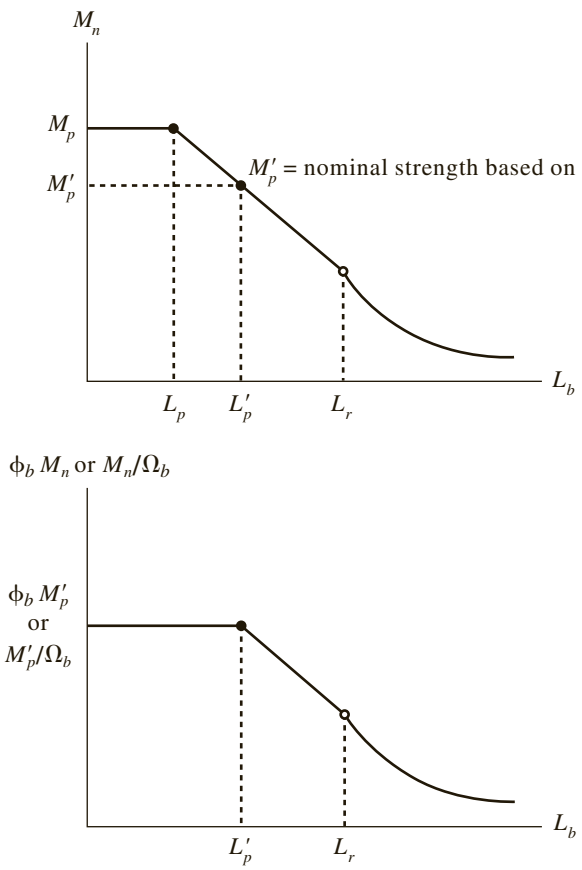
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If a dashed portion of a curve is encountered, then a curve for a lighter shape lies above or to the right of the dashed curve. Points on the curves corresponding to L_p are indicated by a solid circle; L_r is represented by an open circle.

In the LRFD solution of Example 5.10, the required design strength was 810.0 ft-kips, and there was continuous lateral support. For continuous lateral support, L_b can be taken as zero. From the charts, the first solid curve above the 810.0 ft-kip mark is for a W24 × 84, the same as selected in Example 5.10. Although $L_b = 0$ is not on this particular chart, the smallest value of L_b shown is less than L_p for all shapes on that page.

The beam curve shown in Figure 5.25 is for a compact shape, so the value of M_n for sufficiently small values of L_b is M_p. As discussed in Section 5.6, if the shape is noncompact, the maximum value of M_n will be based on flange local buckling. The maximum unbraced length for which this condition is true will be different from the value of L_p obtained with AISC Equation F2-5. The moment strength of noncompact shapes is illustrated graphically in Figure 5.27, where the maximum nominal strength is denoted M'_p, and the maximum unbraced length for which this strength is valid is denoted L'_p.

FIGURE 5.27



Form of the strength curve in the charts

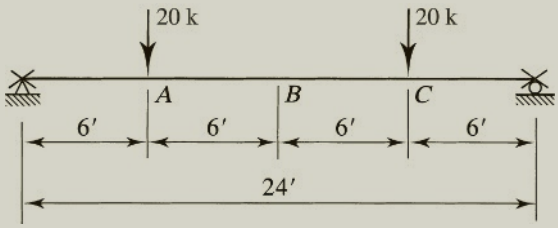
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Although the charts for compact and noncompact shapes are similar in appearance, M_p and L_p are used for compact shapes, whereas M_p' and L_p' are used for noncompact shapes. (This notation is not used in the charts or in any of the other design aids in the *Manual*.) Whether a shape is compact or noncompact is irrelevant to the *use* of the charts.

Example 5.11

The beam shown in Figure 5.28 must support two concentrated *live* loads of 20 kips each at the quarter points. The maximum live load deflection must not exceed $L/240$. Lateral support is provided at the ends of the beam. Use A992 steel and select a W shape.

FIGURE 5.28



Solution

If the weight of the beam is neglected, the central half of the beam is subjected to a uniform moment, and

$$M_A = M_B = M_C = M_{\max} \quad \therefore \; C_b = 1.0$$

Even if the weight is included, it will be negligible compared to the concentrated loads, and C_b can still be taken as 1.0, permitting the charts to be used without modification.

LRFD solution

Temporarily ignoring the beam weight, the factored-load moment is

$$M_u = 6(1.6 \times 20) = 192 \text{ ft-kips}$$

From the charts, with $L_b = 24$ ft, **try W12 × 53:**

$$\phi_b M_n = 209 \text{ ft-kips} > 192 \text{ ft-kips} \qquad \text{(OK)}$$

Now, we account for the beam weight:

$$M_u = 192 + \frac{1}{8}(1.2 \times 0.053)(24)^2 = 197 \text{ ft-kips} < 209 \text{ ft-kips} \qquad \text{(OK)}$$

The shear is

$$V_u = 1.6(20) + \frac{1.2(0.053)(24)}{2} = 32.8 \text{ ft-kips}$$

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From the Z_x table (or the uniform load table),

$$\phi_v V_n = 125 \text{ kips} > 32.8 \text{ kips} \qquad \text{(OK)}$$

The maximum permissible live load deflection is

$$\frac{L}{240} = \frac{24(12)}{240} = 1.20 \text{ in.}$$

From Table 3-23, "Shears, Moments, and Deflections," in Part 3 of the *Manual,* the maximum deflection (at midspan) for two equal and symmetrically placed loads is

$$\Delta = \frac{Pa}{24EI}(3L^2 - 4a^2)$$

where

P = magnitude of concentrated load
a = distance from support to load
L = span length

$$\Delta = \frac{20(6 \times 12)}{24EI}[3(24 \times 12)^2 - 4(6 \times 12)^2] = \frac{13.69 \times 10^6}{EI}$$

$$= \frac{13.69 \times 10^6}{29{,}000(425)} = 1.11 \text{ in.} < 1.20 \text{ in.} \qquad \text{(OK)}$$

Answer Use a W12 × 53.

ASD solution The required flexural strength (not including the beam weight) is

$$M_a = 6(20) = 120 \text{ ft-kips}$$

From the charts, with $L_b = 24$ ft, **try W12 × 53.**

$$\frac{M_n}{\Omega_b} = 139 \text{ ft-kips} > 120 \text{ ft-kips} \qquad \text{(OK)}$$

Account for the beam weight:

$$M_a = 6(20) + \frac{1}{8}(0.053)(24)^2 = 124 \text{ ft-kips} < 139 \text{ ft-kips} \qquad \text{(OK)}$$

The required shear strength is

$$V_a = 20 + \frac{0.053(24)}{2} = 20.6 \text{ kips}$$

From the Z_x table (or the uniform load table),

$$\frac{V_n}{\Omega_v} = 83.2 \text{ kips} > 20.6 \text{ kips} \qquad \text{(OK)}$$

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Since deflections are computed with service loads, the deflection check is the same for both LRFD and ASD. From the LRFD solution,

$$\Delta = 1.11 \text{ in.} < 1.20 \text{ in.} \qquad \text{(OK)}$$

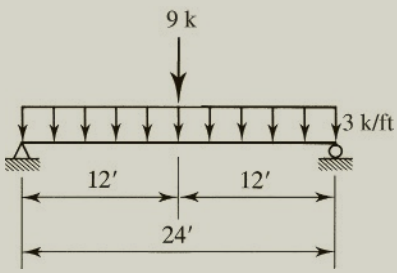
Answer Use a W12 × 53.

Although the charts are based on $C_b = 1.0$, they can easily be used for design when C_b is not 1.0; simply divide the required strength by C_b before entering the charts. We illustrate this technique in Example 5.12.

Example 5.12

Use A992 steel and select a rolled shape for the beam in Figure 5.29. The concentrated load is a service live load, and the uniform load is 30% dead load and 70% live load. Lateral bracing is provided at the ends and at midspan. There is no restriction on deflection.

FIGURE 5.29



Solution Neglect the beam weight and check it later.

$$w_D = 0.30(3) = 0.9 \text{ kips/ft}$$
$$w_L = 0.70(3) = 2.1 \text{ kips/ft}$$

LRFD solution

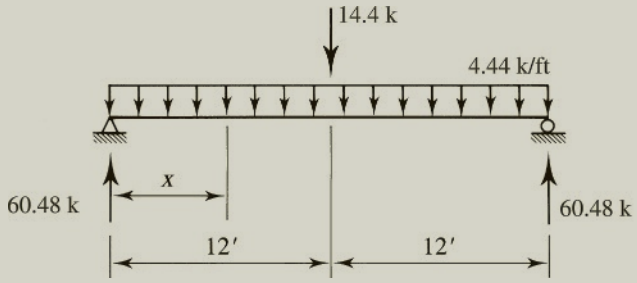
$$w_u = 1.2(0.9) + 1.6(2.1) = 4.44 \text{ kips/ft}$$
$$P_u = 1.6(9) = 14.4 \text{ kips}$$

The factored loads and reactions are shown in Figure 5.30. Next, determine the moments required for the computation of C_b. The bending moment at a distance x from the left end is

$$M = 60.48x - 4.44x\left(\frac{x}{2}\right) = 60.48x - 2.22x^2 \qquad (\text{for } x \le 12 \text{ ft})$$

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FIGURE 5.30



For $x = 3$ ft, $M_A = 60.48(3) - 2.22(3)^2 = 161.5$ ft-kips
For $x = 6$ ft, $M_B = 60.48(6) - 2.22(6)^2 = 283.0$ ft-kips
For $x = 9$ ft, $M_C = 60.48(9) - 2.22(9)^2 = 364.5$ ft-kips
For $x = 12$ ft, $M_{\max} = M_u = 60.48(12) - 2.22(12)^2 = 406.1$ ft-kips

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(406.1)}{2.5(406.1) + 3(161.5) + 4(283.0) + 3(364.5)} = 1.36$$

Enter the charts with an unbraced length $L_b = 12$ ft and a bending moment of

$$\frac{M_u}{C_b} = \frac{406.1}{1.36} = 299 \text{ ft-kips}$$

Try W21 × 48:

$\phi_b M_n = 311$ ft-kips (for $C_b = 1$)

Since $C_b = 1.36$, the actual design strength is $1.36(311) = 423$ ft-kips. But the design strength cannot exceed $\phi_b M_p$, which is only 398 ft-kips (obtained from the chart), so the actual design strength must be taken as

$\phi_b M_n = \phi_b M_p = 398$ ft-kips $< M_u = 406.1$ ft-kips (N.G.)

For the next trial shape, move up in the charts to the next solid curve and **try W18 × 55**. For $L_b = 12$ ft, the design strength from the chart is 335 ft-kips for $C_b = 1$. The strength for $C_b = 1.36$ is

$\phi_b M_n = 1.36(335) = 456$ ft-kips $> \phi_b M_p = 420$ ft-kips

$\therefore\ \phi_b M_n = \phi_b M_p = 420$ ft-kips $> M_u = 406.1$ ft-kips (OK)

Check the beam weight.

$$M_u = 406.1 + \frac{1}{8}(1.2 \times 0.055)(24)^2 = 411 \text{ ft-kips} < 420 \text{ ft-kips} \quad \text{(OK)}$$

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The maximum shear is

$$V_u = 60.48 + \frac{1.2(0.055)}{2}(24) = 61.3 \text{ kips}$$

From the Z_x tables,

$$\phi_v V_n = 212 \text{ kips} > 61.3 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W18 × 55.

ASD solution The applied loads are

$$w_a = 3 \text{ kips/ft} \quad \text{and} \quad P_a = 9 \text{ kips}$$

The left-end reaction is

$$\frac{w_a L + P_a}{2} = \frac{3(24) + 9}{2} = 40.5 \text{ kips}$$

and the bending moment at a distance x from the left end is

$$M = 40.5x - 3x\left(\frac{x}{2}\right) = 40.5x - 1.5x^2 \qquad (\text{for } x \le 12 \text{ ft})$$

Compute the moments required for the computation of C_b:

For $x = 3$ ft, $M_A = 40.5(3) - 1.5(3)^2 = 108.0$ ft-kips
For $x = 6$ ft, $M_B = 40.5(6) - 1.5(6)^2 = 189.0$ ft-kips
For $x = 9$ ft, $M_C = 40.5(9) - 1.5(9)^2 = 243.0$ ft-kips
For $x = 12$ ft, $M_{\max} = 40.5(12) - 1.5(12)^2 = 270.0$ ft-kip

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(270)}{2.5(270) + 3(108) + 4(189) + 3(243)} = 1.36$$

Enter the charts with an unbraced length $L_b = 12$ ft and a bending moment of

$$\frac{M_a}{C_b} = \frac{270}{1.36} = 199 \text{ ft-kips}$$

Try W21 × 48: For $C_b = 1$,

$$M_n/\Omega_b = 207 \text{ ft-kips}$$

For $C_b = 1.36$, the actual allowable strength is 1.36(207) = 282 ft-kips, but the strength cannot exceed M_p/Ω_n, which is only 265 ft-kips (this can be obtained from the chart), so the actual allowable strength must be taken as

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 265 \text{ ft-kips} < M_a = 270 \text{ ft-kips} \qquad \text{(N.G.)}$$

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Move up in the charts to the next solid curve and **try W18 × 55**. For $L_b = 12$ ft, the allowable strength for $C_b = 1$ is 223 ft-kips. The strength for $C_b = 1.36$ is

$$\frac{M_n}{\Omega_b} = 1.36(223) = 303 \text{ ft-kips} > \frac{M_p}{\Omega_b} = 280 \text{ ft-kips}$$

$$\therefore \quad \frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 280 \text{ ft-kips} > M_a = 270 \text{ ft-kips} \qquad \text{(OK)}$$

Account for the beam weight.

$$M_a = 270 + \frac{1}{8}(0.055)(24)^2 = 274 \text{ ft-kips} < 280 \text{ ft-kips} \qquad \text{(OK)}$$

The maximum shear is

$$V_a = \frac{9 + 3.055(24)}{2} = 41.2 \text{ kips}$$

From the Z_x table (or the uniform load table)

$$\frac{V_n}{\Omega_v} = 141 \text{ kips} > 41.2 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W18 × 55.

In Example 5.12, the value of C_b is the same (to three significant figures) for both the factored and the unfactored moments. The two computed values will always be nearly the same, and for this reason, it makes no practical difference which moments are used.

If deflection requirements control the design of a beam, a minimum required moment of inertia is computed, and the lightest shape having this value is sought. This task is greatly simplified by the moment of inertia selection tables in Part 3 of the *Manual*. We illustrate the use of these tables in Example 5.15, following a discussion of the design procedure for a beam in a typical floor or roof system.

Manual Table 6-2 for Beam Analysis and Design

Manual Table 6-2 was discussed in Chapter 4 in connection with compressive strength. This table also can be used to determine the flexural and shear strengths of W shapes with $F_y = 50$ ksi. The table includes all W shapes, including those not covered in other tables or curves, so it can be used in lieu of those other design aids (for $F_y = 50$ ksi).

Most pages of the table cover three shapes. The headings for each shape are listed twice; once on the left side of the page and once on the right side. The right side is used to determine the available flexural strength, M_{nx}/Ω_b or $\phi_b M_{nx}$. To obtain the strength, enter the middle column with the unbraced length, L_b. In the corresponding row, the flexural strength for bending about the x axis is given below

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the shape designation. All strength values account for local buckling for noncompact shapes.

The strength values correspond to $C_b = 1$. For other values of C_b, multiply the tabulated values by C_b, just as you would when using the beam design curves. To be sure that this increased strength does not exceed M_{px}/Ω_b or $\phi_b M_{px}$, compare it with the strength corresponding to $L_b = 0$. This strength corresponds to M_{px}/Ω_b or $\phi_b M_{px}$. The increased strength should not exceed this value.

Other useful values are given at the bottom of the table. On the left side, you will find the available strength in tension yielding, the available strength in tension rupture (based on $A_e = 0.75A_g$), the available strength in shear, and the available flexural strength for bending about the y axis. On the right side, values include L_p, L_r, A_g, I_x, I_y, and r_y.

The use of *Manual* Table 6-2 is illustrated in Examples 5.13 and 5.14.

Example 5.13

A W12 × 50 of A992 steel has a simply supported span length of 14 ft with lateral bracing only at the ends and at midspan. There is a concentrated live load of 40 kips at midspan. The only dead load is the beam weight. Use *Manual* Table 6-2 and determine whether this beam has enough strength.

Solution Use Figure 5.15 to determine C_b. Since the dead load is so much smaller than the live load, we can approximate the loading as shown in Figure 5.15b. Therefore, use $C_b = 1.67$.

LRFD solution

$$P_u = 1.6(40) = 64 \text{ kips}, \quad w_u = 1.2(0.050) = 0.060 \text{ kips/ft}$$

$$M_u = \frac{w_u L^2}{8} + \frac{P_u L}{4} = \frac{0.060(14)^2}{8} + \frac{64(14)}{4} = 226 \text{ ft-kips}$$

From *Manual* Table 6-2, for $L_b = 7$ ft,

$$\phi_b M_{nx} = 269 \text{ ft-kips} > 226 \text{ ft-kips} \qquad \text{(OK)}$$

Since this beam is adequate in flexure with $C_b = 1$, it will be adequate with $C_b = 1.67$ (or any other value, since the minimum value of C_b is 1).

Check shear: From *Manual* Table 6-2, the available shear strength is

$$\phi_v V_n = 135 \text{ kips}$$

The required shear strength is

$$V_u = \frac{P_u}{2} + \frac{w_u L}{2} = \frac{64}{2} + \frac{0.060(14)}{2} = 32.4 \text{ kips} < 135 \text{ kips} \qquad \text{(OK)}$$

Answer The beam has enough strength.

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ASD solution

$P_a = 40$ kips, $w_a = 0.050$ kips/ft

$$M_a = \frac{w_a L^2}{8} + \frac{P_a L}{4} = \frac{0.050(14)^2}{8} + \frac{40(14)}{4} = 141 \text{ ft-kips}$$

From *Manual* Table 6-2, for $L_b = 7$ ft,

$M_{nx}/\Omega_b = 179$ ft-kips > 141 ft-kips (OK)

Since this beam is adequate in flexure with $C_b = 1$, it will be adequate with $C_b = 1.67$.

Check shear: From *Manual* Table 6-2, the available shear strength is

$$\frac{V_n}{\Omega_v} = 90.3 \text{ kips}$$

The required shear strength is

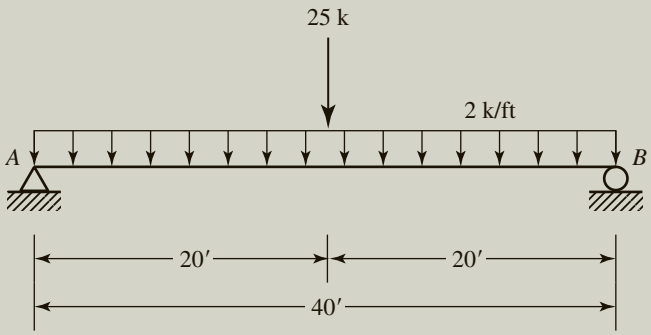
$$V_a = \frac{P_a}{2} + \frac{w_a L}{2} = \frac{40}{2} + \frac{0.050(14)}{2} = 20.3 \text{ kips} < 90.3 \text{ kips} \quad \text{(OK)}$$

Answer The beam has enough strength.

Example 5.14

The beam shown in Figure 5.31 has lateral support only at the ends. The uniform load is a superimposed dead load, and the concentrated load is a live load. Use A992 steel and select a W shape. The live load deflection must not exceed $L/360$. Use LRFD.

FIGURE 5.31



Solution $w_u = 1.2w_D + 1.6w_L = 1.2(2) = 2.4$ kips/ft (neglect beam weight and check it later)

$P_u = 1.2P_D + 1.6P_L = 1.6(25) = 40$ kips

$$\text{Left reaction} = \frac{40 + 2.4(40)}{2} = 68.0 \text{ kips}$$

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For $L_b = 40$ ft, compute C_b by dividing the 40-foot unbraced length into four 10-foot segments. This places A, B, and C at distances of 10, 20, and 30 feet from the left end, respectively.

$$M_A = M_C = 68(10) - 2.4(10)^2/2 = 560.0 \text{ ft-kips}$$
$$M_B = M_{\max} = 68(20) - 2.4(20)^2/2 = 880.0 \text{ ft-kips}$$
$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$
$$= \frac{12.5(880)}{2.5(880) + 3(560) + 4(880) + 3(560)} = 1.211$$

Enter *Manual* Table 6-2 with $L_b = 40$ ft and search along this same row for a flexural strength of at least

$$\frac{\phi_b M_{nx}}{C_b} = \frac{\phi_b M_{\max}}{C_b} = \frac{880.0}{1.211} = 727 \text{ ft-kips}$$

Try a W24 × 146, with $\phi_b M_{nx}/C_b = 771$ ft-kips.

For $C_b = 1.211$,

$$\phi_b M_{nx} = 1.211(771) = 934 \text{ ft-kips} > 880 \text{ ft-kips} \qquad \text{(OK)}$$

For an unbraced length of zero,

$$\phi_b M_{nx} = \phi_b M_{px} = 1570 \text{ ft-kips} < 934 \text{ ft-kips} \qquad \text{(OK)}$$

Check beam weight:

$$M_u = 880 + \frac{1}{8}(1.2 \times 0.146)(40)^2 = 915 \text{ ft-kips} < 1570 \text{ ft-kips} \qquad \text{(OK)}$$

Check deflection:

$$\text{Maximum } \Delta_L = \frac{L}{360} = \frac{40(12)}{360} = 1.33 \text{ in.}$$

From the bottom right side of *Manual* Table 6-2, $I_x = 4580 \text{ in}^4$, so

$$\Delta_L = \frac{P_L L^3}{48EI} = \frac{25(40 \times 12)^3}{48(29{,}000)(4580)} = 0.434 \text{ in.} < 1.33 \text{ in.} \qquad \text{(OK)}$$

Check shear: From the bottom left side of Table 6-2, $\phi_v V_n = 482$ kips, so

$$V_u = \frac{40}{2} + \frac{1.2(2.4 + 0.146)(40)}{2} = 81.1 \text{ kips} < 482 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W24 × 146.

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5.11 FLOOR AND ROOF FRAMING SYSTEMS

When a distributed load acts on an area such as a floor in a building, certain portions of that load are supported by various components of the floor system. The actual distribution is difficult to determine, but it can be approximated quite easily. The basic idea is that of *tributary areas*. In the same way that tributaries flow into a river and contribute to the volume of water in it, the loads on certain areas of a structural surface "flow" into a structural component. The concept of tributary areas was first discussed in Section 3.8 in the coverage of tension members in roof trusses.

Figure 5.32 shows a typical floor framing plan for a multistory building. Part (a) of the figure shows one of the rigid frames comprising the building. Part (b) shows what would be seen if a horizontal section were cut through the building above one of the floors and the lower portion viewed from above. The gridwork thus exposed consists of the column cross sections (in this case, wide-flange structural steel shapes), girders connecting the columns in the east-west direction, and intermediate floor beams such as *EF* spanning between the girders. Girders are defined as beams that support other beams, although sometimes the term is applied to large beams in general. The floor beams, which fill in the panels defined by the columns, are sometimes called *filler beams*. The columns and girders along any of the east-west lines make up an individual frame. The frames are connected by the beams in the north-south direction, completing the framework for the building. There may also be secondary components, such as bracing for stability, that are not shown in Figure 5.32.

Figure 5.32c shows a typical bay of the floor framing system. When columns are placed in a rectangular grid, the region between four columns is called a *bay*. The bay size, such as 30 ft $\times$ 40 ft, is a measure of the geometry of a building. Figure 5.32d is a cross section of this bay, showing the floor beams as wide-flange steel shapes supporting a reinforced concrete floor slab.

The overall objective of a structure is to transmit loads to the foundation. As far as floor loads are concerned, this transmission of loads is accomplished as follows:

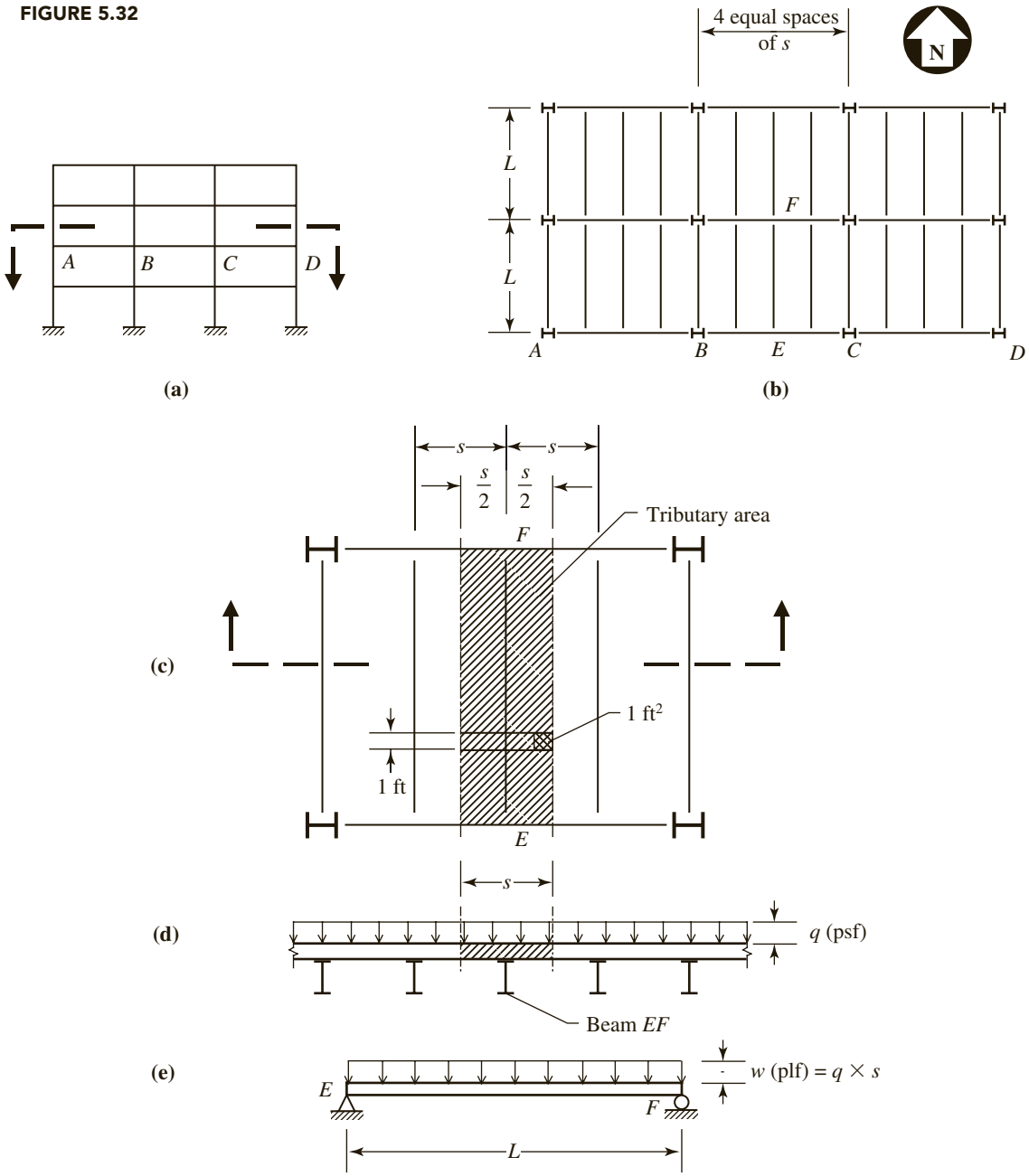
1. Floor loads, both live and dead, are supported by the floor slab.
2. The weight of the slab, along with the loads it supports, is supported by the floor beams.
3. The floor beams transmit their loads, including their own weight, to the girders.
4. The girders and their loads are supported by the columns.
5. The column loads are supported by the columns of the story below. The column loads accumulate from the top story to the foundation.

(The route taken by the loads from one part of the structure to another is sometimes called the *load path*.) This is a fairly accurate representation of what happens, but it is not exact. For example, part of the slab and its load will be supported directly by the girders, but most of it will be carried by the floor beams.

Figure 5.32c shows a shaded area around floor beam *EF*. This is the tributary area for this member, and it consists of half the floor between beam *EF* and the adjacent beam on each side. Thus, the total width of floor being supported is equal to the beam spacing *s* if the spacing is uniform. If the load on the floor is uniformly distributed,

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FIGURE 5.32



we can express the uniform load on beam *EF* as a force per unit length (for example, pounds per linear foot [plf]) by multiplying the floor load in force per unit area (for example, pounds per square foot [psf]) by the tributary width *s*. Figure 5.32e shows the final beam model (for the usual floor framing connections, the beams can be treated as simply supported).

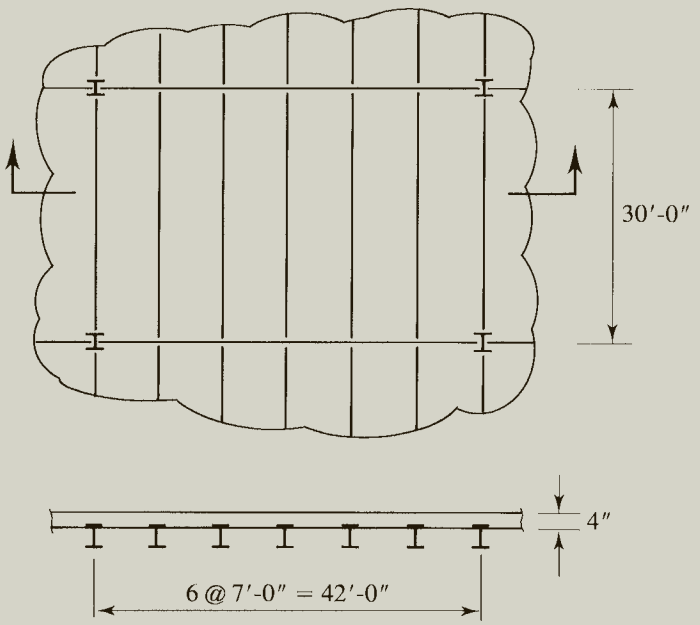
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For convenience, the weight of a reinforced concrete floor slab is usually expressed in pounds per square foot of floor surface. This way, the slab weight can be combined with other loads similarly expressed. If the floor consists of a metal deck and concrete fill, the combined weight can usually be obtained from the deck manufacturer's literature. If the floor is a slab of uniform thickness, the weight can be calculated as follows. Normal-weight concrete weighs approximately 145 pounds per cubic foot. If 5 pcf is added to account for the reinforcing steel, the total weight is 150 pcf. The volume of slab contained in one square foot of floor is 1 ft^2 × the slab thickness t. For a thickness expressed in inches, the slab weight is therefore $(t/12)$ (150) psf. For lightweight concrete, a unit weight of 115 pounds per cubic foot can be used in lieu of more specific data.

Example 5.15

Part of a floor framing system is shown in Figure 5.33. A 4-inch-thick reinforced concrete floor slab of normal-weight concrete is supported by floor beams spaced at 7 feet. The floor beams are supported by girders, which in turn are supported by the columns. In addition to the weight of the structure, loads consist of a uniform live load of 80 psf and moveable partitions, to be accounted for by using a uniformly distributed load of 20 pounds per square foot of floor surface. The maximum live load deflection must not exceed 1/360 of the span length. Use A992 steel and design the floor beams. Assume that the slab provides continuous lateral support of the floor beams.

FIGURE 5.33



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Solution The slab weight is

$$w_{\text{slab}} = \frac{t}{12}(150) = \frac{4}{12}(150) = 50 \text{ psf}$$

Assume that each beam supports a 7-ft width (tributary width) of floor.

Slab: $50(7) = 350$ lb/ft
Partitions: $20(7) = 140$ lb/ft
Live load: $80(7) = 560$ lb/ft

The beam weight will be accounted for once a trial selection has been made.

Since the partitions are moveable, they will be treated as live load. This is consistent with the provisions of the International Building Code (ICC, 2015). The dead and live loads are, therefore,

$w_D = 0.350$ lb/ft (excluding the beam weight)
$w_L = 0.560 + 0.140 = 0.700$ lb/ft

LRFD solution The total factored load is

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.350) + 1.6(0.700) = 1.540 \text{ kips/ft}$$

The typical floor beam connection will provide virtually no moment restraint, and the beams can be treated as simply supported. Therefore,

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(1.540)(30)^2 = 173 \text{ ft-kips}$$

The maximum permissible live-load deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1.0 \text{ in.}$$

From $\Delta_L = \frac{5}{384}\frac{w_L L^4}{EI}$, the required moment of inertia is

$$I_{\text{required}} = \frac{5w_L L^4}{384E\Delta_{\text{required}}} = \frac{5(0.700/12)(30 \times 12)^4}{384(29{,}000)(1.0)} = 440 \text{ in.}^4$$

Since the beams have continuous lateral support, the Z_x table can be used to select a trial shape that will satisfy the moment requirement.

Try a W14 × 30:

$\phi_b M_n = 177$ ft-kips > 173 ft-kips (OK)

Check the beam weight:

$$M_u = 173 + \frac{1}{8}(1.2 \times 0.030)(30)^2 = 177 \text{ ft-kips} \quad \text{(OK)}$$

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Part 3 of the *Manual* contains selection tables for both I_x and I_y. These tables are organized in the same way as the Z_x table, so selection of the lightest shape with sufficient moment of inertia is simple. From the I_x table, we see that a W18 × 35 is the lightest shape that satisfies our requirement.

Try a W18 × 35:

$$I_x = 510 \text{ in.}^4 > 440 \text{ in.}^4 \qquad \text{(OK)}$$

From the Z_x table,

$$\phi_b M_{nx} = 249 \text{ ft-kips} > 177 \text{ ft-kips} \qquad \text{(OK)}$$

The maximum shear is

$$V_u = \frac{w_u L}{2} \approx \frac{1.540(30)}{2} = 23.1 \text{ kips}$$

From the Z_x table, the available shear strength is

$$\phi_v V_n = 159 \text{ kips} > 23.1 \text{ kips} \qquad \text{(OK)}$$

Answer Use a W18 × 35.

ASD solution Account for the beam weight after a selection has been made.

$$w_a = w_D + w_L = 0.350 + 0.700 = 1.05 \text{ kips/ft}$$

If we treat the beam connection as a simple support, the required moment strength is

$$M_a = \frac{1}{8} w_a L^2 = \frac{1}{8}(1.05)(30)^2 = 118 \text{ ft-kips} = \text{required } \frac{M_n}{\Omega_b}$$

The maximum permissible live-load deflection is

$$\frac{L}{360} = \frac{30(12)}{360} = 1.0 \text{ in.}$$

From $\Delta_L = \dfrac{5}{384} \dfrac{w_L L^4}{EI}$, the required moment of inertia is

$$I_{\text{required}} = \frac{5 w_L L^4}{384 E \Delta_{\text{required}}} = \frac{5(0.700/12)(30 \times 12)^4}{384(29{,}000)(1.0)} = 440 \text{ in.}^4$$

Part 3 of the *Manual* contains selection tables for both I_x and I_y. These tables are organized in the same way as the Z_x table, so selection of the lightest shape with sufficient moment of inertia is simple. From the I_x table, we see that a W18 × 35 is the lightest shape that satisfies our requirement.

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Try a W18 × 35:

$$I_x = 510 \text{ in.}^4 > 440 \text{ in.}^4 \qquad \text{(OK)}$$

$$\frac{M_n}{\Omega_b} = 166 \text{ ft-kips} > 118 \text{ ft-kips} \qquad \text{(OK)}$$

The maximum shear is

$$V_a = \frac{w_a L}{2} = \frac{(1.05 + 0.031)(30)}{2} = 16.2 \text{ kips}$$

From the Z_x table, the available shear strength is

$$\frac{V_n}{\Omega_v} = 87.5 \text{ kips} > 16.2 \text{ kips} \quad \text{(OK)}$$

Answer Use a W18 × 35.

Note that in Example 5.15, the design was controlled by serviceability rather than strength. This is not unusual. Although there is no limit on the dead load deflection in this example, this deflection may be needed if the beam is to be cambered.

$$\Delta_D = \frac{5}{384}\frac{w_{\text{slab + beam}} L^4}{EI} = \frac{5}{384}\frac{[(0.350 + 0.035)/12](30 \times 12)^4}{29{,}000(510)} = 0.474 \text{ in.}$$

5.12 HOLES IN BEAMS

If beam connections are made with bolts, holes will be punched or drilled in the beam web or flanges. In addition, relatively large holes are sometimes cut in beam webs to provide space for utilities such as electrical conduits and ventilation ducts. Ideally, holes should be placed in the web only at sections of low shear, and holes should be made in the flanges at points of low bending moment. That will not always be possible, so the effect of the holes must be accounted for.

For relatively small holes such as those for bolts, the effect will be small, particularly for flexure, for two reasons. First, the reduction in the cross section is usually small. Second, adjacent cross sections are not reduced, and the change in cross section is actually more of a minor discontinuity than a "weak link."

Holes in a beam flange are of concern for the tension flange only, since bolts in the compression flange will transmit the load through the bolts. This is the same rationale that is used for compression members, where the net area is not considered. The AISC Specification requires that bolt holes in beam flanges be accounted for when the nominal tensile rupture strength (fracture strength) of the flange is less than the nominal tensile yield strength—that is, when

$$F_u A_{fn} < F_y A_{fg} \tag{5.9}$$

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where

A_{fn} = net tension flange area
A_{fg} = gross tension flange area

If $F_y/F_u > 0.8$, the Specification requires that the right hand side of Equation 5.9 be increased by 10%. Equation 5.9 can be written more generally as follows:

$$F_u A_{fn} < Y_t F_y A_{fg} \tag{5.10}$$

where

$Y_t = 1.0$ for $F_y/F_u \leq 0.8$
$= 1.1$ for $F_y/F_u > 0.8$

Note that, for A992 steel, the preferred steel for W shapes, the *maximum* value of F_y/F_u is 0.85. This means that unless more information is available, use $Y_t = 1.1$.
If the condition of Equation 5.10 exists—that is, if

$$F_u A_{fn} < Y_t F_y A_{fg}$$

then AISC F13.1 requires that the nominal flexural strength be limited by the condition of flexural rupture. This limit state corresponds to a flexural stress of

$$f_b = \frac{M_n}{S_x(A_{fn}/A_{fg})} = F_u \tag{5.11}$$

where $S_x(A_{fn}/A_{fg})$ can be considered to be a "net" elastic section modulus. The relationship of Equation 5.11 corresponds to a nominal flexural strength of

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x$$

The AISC requirement for holes in beam flanges can be summarized as follows:
If

$$F_u A_{fn} < Y_t F_y A_{fg}$$

The nominal flexural strength cannot exceed

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x \qquad \text{(AISC Equation F13-1)}$$

where

$Y_t = 1.0$ for $F_y/F_u \leq 0.8$
$= 1.1$ for $F_y/F_u > 0.8$

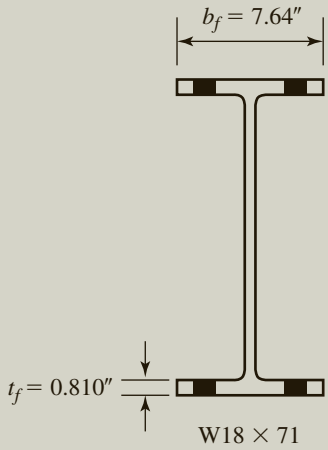
The constant Y_t should be taken as 1.1 for A992 steel or if the maximum value of F_y/F_u is not known.

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Example 5.16

The shape shown in Figure 5.34 is a W18 × 71 with holes in each flange for 1-inch-diameter bolts. The steel is A992. Compute the nominal flexural strength for an unbraced length of 10 feet. Use $C_b = 1.0$.

FIGURE 5.34



Solution To determine the nominal flexural strength M_n, all applicable limit states must be checked. From the Z_x table, a W18 × 71 is seen to be a compact shape (no footnote to indicate otherwise). Also from the Z_x table, $L_p = 6.00$ ft and $L_r = 19.6$ ft. Therefore, for an unbraced length $L_b = 10$ ft,

$$L_p < L_b < L_r$$

and the beam is subject to inelastic lateral-torsional buckling. The nominal strength for this limit state is given by

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(AISC Equation F2-2)}$$

where

$$M_p = F_yZ_x = 50(146) = 7300 \text{ in.-kips}$$

$$M_n = 1.0\left[7300 - (7300 - 0.7 \times 50 \times 127)\left(\frac{10 - 6}{19.6 - 6}\right)\right] = 6460 \text{ in.-kips}$$

Check to see if the flange holes need to be accounted for. The gross area of one flange is

$$A_{fg} = t_fb_f = 0.810(7.64) = 6.188 \text{ in.}^2$$

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For a 1-inch diameter bolt, the standard hole size is

$$d_h = 1 + \frac{1}{8} = 1\frac{1}{8} \text{ in.}$$

The effective hole diameter is

$$d_h + \frac{1}{16} = 1\frac{1}{8} + \frac{1}{16} = 1.188 \text{ in.}$$

and the net flange area is

$$A_{fn} = A_{fg} - t_f \Sigma d_h = 6.188 - 0.810(2 \times 1.188) = 4.263 \text{ in.}^2$$

$$F_u A_{fn} = 65(4.263) = 277.1 \text{ kips}$$

Determine Y_t. For A992 steel, the maximum F_y/F_u ratio is 0.85. Since this is greater than 0.8, use $Y_t = 1.1$.

$$Y_t F_y A_{fg} = 1.1(50)(6.188) = 340.3 \text{ kips}$$

Since $F_u A_{fn} < Y_t F_y A_{fg}$, the holes must be accounted for. From AISC Equation F13-1,

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x = \frac{277.1}{6.188}(127) = 5687 \text{ in.-kips}$$

This value is less than the LTB value of 6460 in.-kips, so it controls.

Answer $M_n = 5687$ in.-kips = 474 ft-kips.

Beams with large holes in their webs will require special treatment and are beyond the scope of this book. *Design of Steel and Composite Beams with Web Openings* is a useful guide to this topic (Darwin, 1990).

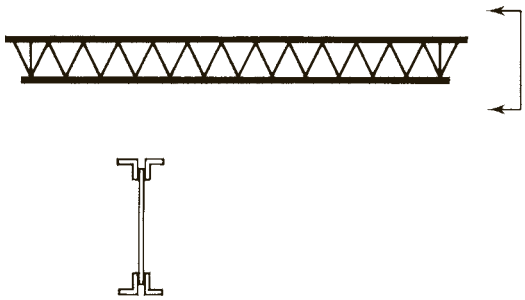
5.13 OPEN-WEB STEEL JOISTS

Open-web steel joists are prefabricated trusses of the type shown in Figure 5.35. Many of the smaller ones use a continuous circular bar to form the web members and are commonly called *bar* joists. They are used in floor and roof systems in a wide variety of structures. For a given span, an open-web joist will be lighter in weight than a rolled shape, and the absence of a solid web allows for the easy passage of duct work and electrical conduits. Depending on the span length, open-web joists may be more economical than rolled shapes, although there are no general guidelines for making this determination.

Open-web joists are available in standard depths and load capacities from various manufacturers. Some open-web joists are designed to function as floor or roof joists, and others are designed to function as girders, supporting the concentrated

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FIGURE 5.35



reactions from joists. The AISC Specification does not cover open-web steel joists; a separate organization, the Steel Joist Institute (SJI), exists for this purpose. All aspects of steel joist usage, including their design and manufacture, are addressed in the publication *Standard Specifications, Load Tables, and Weight Tables for Steel Joists and Joist Girders* (SJI, 2010).

An open-web steel joist can be selected with the aid of the standard load tables (SJI, 2010). These tables give load capacities in pounds per foot of length for various standard joists. Tables are available for both LRFD and ASD, in either U.S. Customary units or metric units. One of the LRFD tables is reproduced in Figure 5.36. For each combination of span and joist, a pair of load values is given. The top number is the total load capacity in pounds per foot. The bottom number is the live load per foot that will produce a deflection of 1/360 of the span length. For span lengths in the shaded areas, special bridging (interconnection of joists) is required. The ASD tables use the same format, but the loads are unfactored. The first number in the designation is the nominal depth in inches. The table also gives the approximate weight in pounds per foot of length. Steel fabricators who furnish open-web steel joists must certify that a particular joist of a given designation, such as a 10K1 of span length 20 feet, will have a safe load capacity of at least the value given in the table. Different manufacturers' 10K1 joists may have different member cross sections, but they all must have a nominal depth of 10 inches and, for a span length of 20 feet, a factored load capacity of at least 361 pounds per foot.

The open-web steel joists that are designed to function as floor or roof joists (in contrast to girders) are available as open-web steel joists (K-series, both standard and KCS), longspan steel joists (LH-series), and deep longspan steel joists (DLH-series). Standard load tables are given for each of these categories. The higher you move up the series, the greater the available span lengths and load-carrying capacities become. At the lower end, a 10K1 is available with a span length of 10 feet and a factored load capacity of 825 pounds per foot, whereas a 72DLH19 can support a load of 745 pounds per foot on a span of 144 feet.

With the exception of the KCS joists, all open-web steel joists are designed as simply supported trusses with uniformly distributed loads on the top chord. This loading subjects the top chord to bending as well as axial compression, so the top chord is designed as a beam–column (see Chapter 6). To ensure stability of the top chord, the floor or roof deck must be attached in such a way that continuous lateral support is provided.

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FIGURE 5.36

LRFD

STANDARD LOAD TABLE FOR OPEN WEB STEEL JOISTS, K-SERIES
Based On A 50 ksi Maximum Yield Strength - Loads Shown In Pounds Per Linear Foot (plf)

Joist Designation	10K1	12K1	12K3	12K5	14K1	14K3	14K4	14K6	16K2	16K3	16K4	16K5	16K6	16K7	16K9
Depth (in.)	10	12	12	12	14	14	14	14	16	16	16	16	16	16	16
Approx. Wt (lbs./ft.)	5.0	5.0	5.7	7.1	5.2	6.0	6.7	7.7	5.5	6.3	7.0	7.5	8.1	8.6	10.0
Span (ft.) ↓															
10	825 550														
11	825 542														
12	825 455	825 550	825 550	825 550											
13	718 363	825 510	825 510	825 510											
14	618 289	750 425	825 463	825 463	825 550	825 550	825 550	825 550							
15	537 234	651 344	814 428	825 434	766 475	825 507	825 507	825 507							
16	469 192	570 282	714 351	825 396	672 390	825 467	825 467	825 467	825 550	825 550	825 550	825 550	825 550	825 550	825 550
17	415 159	504 234	630 291	825 366	592 324	742 404	825 443	825 443	768 488	825 526	825 526	825 526	825 526	825 526	825 526
18	369 134	448 197	561 245	760 317	528 272	661 339	795 397	825 408	684 409	762 456	825 490	825 490	825 490	825 490	825 490
19	331 113	402 167	502 207	681 269	472 230	592 287	712 336	825 383	612 347	682 386	820 452	825 455	825 455	825 455	825 455
20	298 97	361 142	453 177	613 230	426 197	534 246	642 287	787 347	552 297	615 330	739 386	825 426	825 426	825 426	825 426
21		327 123	409 153	555 198	385 170	483 212	582 248	712 299	499 255	556 285	670 333	754 373	822 405	825 406	825 406
22		298 106	373 132	505 172	351 147	439 184	529 215	648 259	454 222	505 247	609 289	687 323	747 351	825 385	825 385
23		271 93	340 116	462 150	321 128	402 160	483 188	592 226	415 194	462 216	556 252	627 282	682 307	760 339	825 363
24		249 81	312 101	423 132	294 113	367 141	442 165	543 199	381 170	424 189	510 221	576 248	627 269	697 298	825 346
25					270 100	339 124	408 145	501 175	351 150	390 167	469 195	529 219	576 238	642 263	771 311
26					249 88	313 110	376 129	462 156	324 133	360 148	433 173	489 194	532 211	592 233	711 276
27					231 79	289 98	349 115	427 139	300 119	334 132	402 155	453 173	493 188	549 208	658 246
28					214 70	270 88	324 103	397 124	279 106	310 118	373 138	421 155	459 168	510 186	612 220
29									259 95	289 106	348 124	391 139	427 151	475 167	570 198
30									241 86	270 96	324 112	366 126	399 137	444 151	532 178
31									226 78	252 87	304 101	342 114	373 124	415 137	498 161
32									213 71	237 79	285 92	321 103	349 112	388 124	466 147

Standard Specifications, Load Tables, and Weight Tables for Steel Joists and Joist Girders. Myrtle Beach, S.C.: Steel Joist Institute, 2005. Reprinted with permission.

KCS joists are designed to support both concentrated loads and distributed loads (including nonuniform distributions). To select a KCS joist, the engineer must compute the maximum moment and shear in the joist and enter the KCS tables with these values. (The KCS joists are designed to resist a uniform moment

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and a constant shear.) If concentrated loads must be supported by an LH or a DLH joist, a special analysis should be requested from the manufacturer.

Both top and bottom chord members of K-series joists must be made of steel with a yield stress of 50 ksi, and the web members may have a yield stress of either 36 ksi or 50 ksi. All members of LH- and DLH-series joists can be made with steel of any yield stress between 36 ksi and 50 ksi inclusive. The load capacity of K-series joists must be verified by the manufacturer by testing. No testing program is required for LH- or DLH-series joists.

Joist girders are designed to support open-web steel joists. For a given span, the engineer determines the number of joist spaces, then from the joist girder weight tables selects a depth of girder. The joist girder is designated by specifying its depth, the number of joist spaces, the load at each loaded top-chord panel point of the joist girder, and a letter to indicate whether the load is factored ("F") or unfactored ("K"). For example, using LRFD and U.S. Customary units, a 52G9N10.5F is 52 inches deep, provides for nine equal joist spaces on the top chord, and will support 10.5 kips of factored load at each joist location. The joist girder weight tables give the weight in pounds per linear foot for the specified joist girder for a specific span length.

Example 5.17

Use the load table given in Figure 5.36 to select an open-web steel joist for the following floor system and loads:

Joist spacing = 3 ft 0 in.
Span length = 20 ft 0 in.

The loads are

3-in. floor slab
Other dead load: 20 psf
Live load: 50 psf

The live load deflection must not exceed $L/360$.

Solution For the dead loads of

$$\text{Slab:} \quad 150\left(\frac{3}{12}\right) = 37.5 \text{ psf}$$

Other dead load: = 20 psf
Joist weight: = 3 psf (est.)
Total: = 60.5 psf

$$w_D = 60.5(3) = 181.5 \text{ lb/ft}$$

For the live load of 50 psf,

$$w_L = 50(3) = 150 \text{ lb/ft}$$

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The factored load is

$$w_u = 1.2w_D + 1.6w_L = 1.2\ (181.5) + 1.6(150) = 458\ \text{lb/ft}$$

Figure 5.36 indicates that the following joists satisfy the load requirement: a 12K5, weighing approximately 7.1 lb/ft; a 14K3, weighing approximately 6.0 lb/ft; and a 16K2, weighing approximately 5.5 lb/ft. No restriction was placed on the depth, so we choose the lightest joist, a 16K2.

To limit the live load deflection to $L/360$, the live load must not exceed

$$297\ \text{lb/ft} > 150\ \text{lb/ft} \quad \text{(OK)}$$

Answer Use a 16K2.

The standard load tables also include a K-series economy table, which facilitates the selection of the lightest joist for a given load.

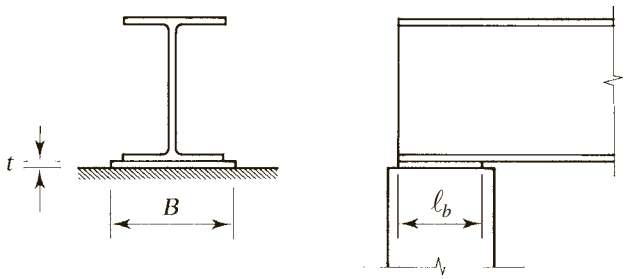
5.14 BEAM BEARING PLATES AND COLUMN BASE PLATES

The design procedure for column base plates is similar to that for beam bearing plates, and for that reason we consider them together. In addition, the determination of the thickness of a column base plate requires consideration of flexure, so it logically belongs in this chapter rather than in Chapter 4. In both cases, the function of the plate is to distribute a concentrated load to the supporting material.

Two types of beam bearing plates are considered: one that transmits the beam reaction to a support such as a concrete wall and one that transmits a load to the top flange of a beam. Consider first the beam support shown in Figure 5.37. Although many beams are connected to columns or other beams, the type of support shown here is occasionally used, particularly at bridge abutments. The design of the bearing plate consists of three steps.

1. Determine dimension ℓ_b so that web yielding and web crippling are prevented.
2. Determine dimension B so that the area $B \times \ell_b$ is sufficient to prevent the supporting material (usually concrete) from being crushed in bearing.
3. Determine the thickness t so that the plate has sufficient bending strength.

FIGURE 5.37



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Web yielding, web crippling, and concrete bearing strength are addressed by AISC in Chapter J, "Design of Connections."

Web Yielding

Web yielding is the compressive crushing of a beam web caused by the application of a compressive force to the flange directly above or below the web. This force could be an end reaction from a support of the type shown in Figure 5.37, or it could be a load delivered to the top flange by a column or another beam. Yielding occurs when the compressive stress on a horizontal section through the web reaches the yield point. When the load is transmitted through a plate, web yielding is assumed to take place on the nearest section of width t_w. In a rolled shape, this section will be at the toe of the fillet, a distance k from the outside face of the flange (this dimension is tabulated in the dimensions and properties tables in the *Manual*). If the load is assumed to distribute itself at a slope of 1 : 2.5, as shown in Figure 5.38, the area at the support subject to yielding is $t_w(2.5k + \ell_b)$. Multiplying this area by the yield stress gives the nominal strength for web yielding at the support:

$$R_n = F_y t_w(2.5k + \ell_b) \qquad \text{(AISC Equation J10-3)}$$

The bearing length ℓ_b at the support should not be less than k.

At the interior load, the length of the section subject to yielding is

$$2(2.5k) + \ell_b = 5k + \ell_b$$

and the nominal strength is

$$R_n = F_y t_w(5k + \ell_b) \qquad \text{(AISC Equation J10-2)}$$

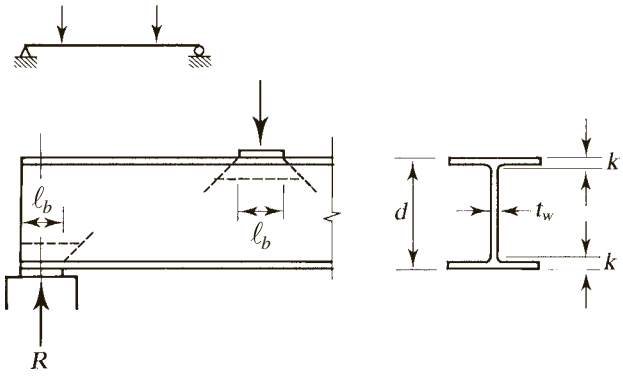
For LRFD, the design strength is ϕR_n, where $\phi = 1.0$.
For ASD, the allowable strength is R_n/Ω, where $\Omega = 1.50$.

Web Crippling

Web crippling is buckling of the web caused by the compressive force delivered through the flange. For an interior load, the nominal strength for web crippling is

$$R_n = 0.80t_w^2\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}}\,Q_f \qquad \text{(AISC Equation J10-4)}$$

FIGURE 5.38



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where

d = total column depth
Q_f = 1.0 for wide-flange shapes
= value from Table K3.2 for HSS

For a load at or near the support (no greater than half the beam depth from the end), the nominal strength is

$$R_n = 0.40t_w^2\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}}Q_f \quad \text{for } \frac{\ell_b}{d} \le 0.2$$

(AISC Equation J10-5a)

or

$$R_n = 0.40t_w^2\left[1 + \left(\frac{4\ell_b}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_yt_f}{t_w}}Q_f \quad \text{for } \frac{\ell_b}{d} > 0.2$$

(AISC Equation J10-5b)

The resistance factor for this limit state is $\phi = 0.75$. The safety factor is $\Omega = 2.00$.

Concrete Bearing Strength

The material used for a beam support can be concrete, brick, or some other material, but it usually will be concrete. This material must resist the bearing load applied by the steel plate. The nominal bearing strength specified in AISC J8 is the same as that given in the American Concrete Institute's Building Code (ACI, 2014) and may be used if no other building code requirements are in effect. If the plate covers the full area of the support, the nominal strength is

$$P_p = 0.85f_c'A_1 \qquad \text{(AISC Equation J8-1)}$$

If the plate does not cover the full area of the support,

$$P_p = 0.85f_c'A_1\sqrt{\frac{A_2}{A_1}} \le 1.7f_c'A_1 \qquad \text{(AISC Equation J8-2)}$$

where

f_c' = 28-day compressive strength of the concrete
A_1 = bearing area
A_2 = full area of the support

If area A_2 is not concentric with A_1, then A_2 should be taken as the largest concentric area that is geometrically similar to A_1, as illustrated in Figure 5.39.

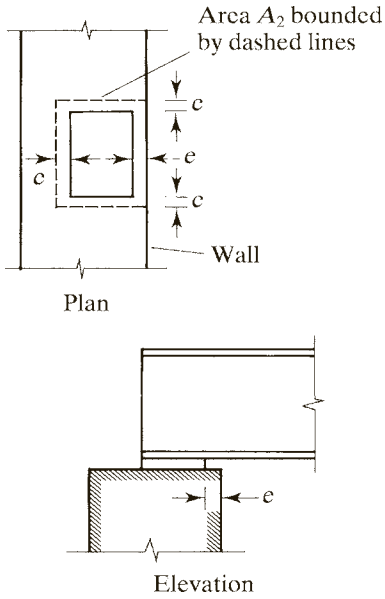
For LRFD, the design bearing strength is ϕ_cP_p, where $\phi_c = 0.65$. For ASD, the allowable bearing strength is P_p/Ω_c, where $\Omega_c = 2.31$.

Plate Thickness

Once the length and width of the plate have been determined, the average bearing pressure is treated as a uniform load on the bottom of the plate, which is assumed to be

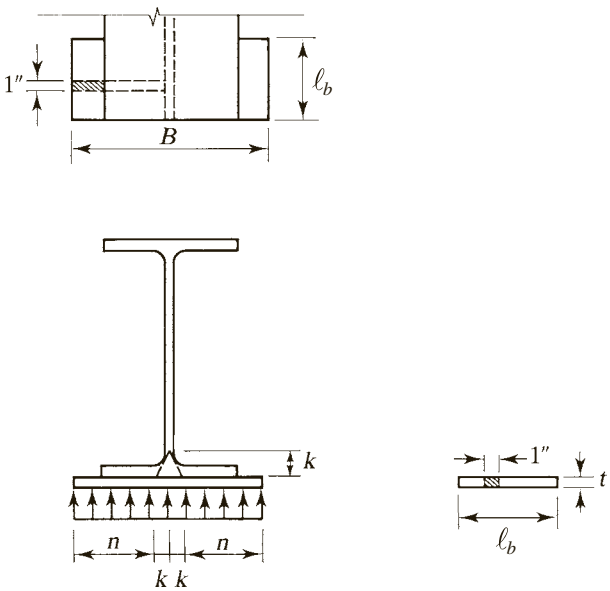
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FIGURE 5.39



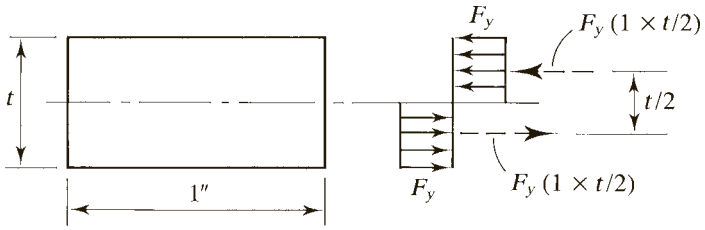
supported at the top over a central width of $2k$ and length ℓ_b, as shown in Figure 5.40. The plate is then considered to bend about an axis parallel to the beam span. Thus the plate is treated as a cantilever of span length $n = (B - 2k)/2$ and a width of ℓ_b. For convenience, a 1-inch width is considered, with a uniform load in pounds per linear inch numerically equal to the bearing pressure in pounds per square inch.

FIGURE 5.40



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FIGURE 5.41



From Figure 5.40, the maximum bending moment in the plate is

$$M = \frac{R}{B\ell_b} \times n \times \frac{n}{2} = \frac{Rn^2}{2B\ell_b}$$

where R is the beam reaction and $R/B\ell_b$ is the average bearing pressure between the plate and the concrete. For a rectangular cross section bent about the minor axis, the nominal moment strength M_n is equal to the plastic moment capacity M_p. As illustrated in Figure 5.41, for a rectangular cross section of unit width and depth t, the plastic moment is

$$M_p = F_y\left(1 \times \frac{t}{2}\right)\left(\frac{t}{2}\right) = F_y\frac{t^2}{4}$$

For LRFD: Since the design strength must at least equal the factored-load moment,

$$\begin{aligned} \phi_b M_p &\geq M_u \\ 0.90F_y\frac{t^2}{4} &\geq \frac{R_u n^2}{2B\ell_b} \\ t &\geq \sqrt{\frac{2R_u n^2}{0.90B\ell_b F_y}} \end{aligned} \tag{5.12}$$

or

$$t \geq \sqrt{\frac{2.2R_u n^2}{B\ell_b F_y}} \tag{5.13}$$

where R_u is the factored-load beam reaction.

For ASD: The allowable flexural strength must at least equal the applied moment:

$$\begin{aligned} \frac{M_p}{\Omega_b} &\geq M_a \\ \frac{F_y t^2/4}{1.67} &\geq \frac{R_a n^2}{2B\ell_b} \\ t &\geq \sqrt{\frac{3.34R_a n^2}{B\ell_b F_y}} \end{aligned} \tag{5.14}$$

where R_a is the service-load beam reaction.

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Example 5.18

Design a bearing plate to distribute the reaction of a W21 × 68 with a span length of 15 feet 10 inches center-to-center of supports. The total service load, including the beam weight, is 9 kips/ft, with equal parts dead and live load. The beam is to be supported on reinforced concrete walls with $f'_c = 3500$ psi. For the beam, $F_y = 50$ ksi, and $F_y = 36$ ksi for the plate.

LRFD solution

The factored load is

$$w_u = 1.2w_D + 1.6w_L = 1.2(4.5) + 1.6(4.5) = 12.60 \text{ kips/ft}$$

and the reaction is

$$R_u = \frac{w_u L}{2} = \frac{12.60(15.83)}{2} = 99.73 \text{ kips}$$

Determine the length of bearing ℓ_b required to prevent web yielding. From AISC Equation J10-3, the nominal strength for this limit state is

$$R_n = F_y t_w (2.5k + \ell_b)$$

For $\phi R_n \geq R_u$,

$$1.0(50)(0.430)[2.5(1.19) + \ell_b] \geq 99.73$$

resulting in the requirement

$$\ell_b \geq 1.66 \text{ in.}$$

(Note that two values of k are given in the dimensions and properties tables: a decimal value, called the *design* dimension, and a fractional value, called the *detailing* dimension. We always use the design dimension in calculations.)

Use AISC Equation J10-5 to determine the value of ℓ_b required to prevent web crippling. Assume $\ell_b/d > 0.2$ and try the second form of the equation, J10-5(b). For $\phi R_n \geq R_u$,

$$\phi(0.40)t_w^2\left[1 + \left(\frac{4\ell_b}{d} - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_u$$

$$0.75(0.40)(0.430)^2\left[1 + \left(\frac{4\ell_b}{21.1} - 0.2\right)\left(\frac{0.430}{0.685}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.685)}{0.430}} \geq 99.73$$

This results in the requirement

$$\ell_b \geq 3.00 \text{ in.}$$

Check the assumption:

$$\frac{\ell_b}{d} = \frac{3.00}{21.1} = 0.14 < 0.2 \qquad \text{(N.G.)}$$

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For $\ell_b/d \le 0.2$,

$$\phi(0.40)t_w^2\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \ge R_u \qquad \text{[AISC Equation J 10-5(a)]}$$

$$0.75(0.40)(0.430)^2\left[1 + 3\left(\frac{\ell_b}{21.1}\right)\left(\frac{0.430}{0.685}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.685)}{0.430}} \ge 99.73$$

resulting in the requirement

$$\ell_b \ge 2.59 \text{ in.}$$

and

$$\frac{\ell_b}{d} = \frac{2.59}{21.1} = 0.12 < 0.2 \qquad \text{(OK)}$$

Try ℓ_b = 6 in: Determine dimension B from a consideration of bearing strength. If we conservatively assume that the full area of the support is used, the required plate area A_1 can be found as follows:

$$\phi_c P_p \ge R_u$$

From AISC Equation J8-1, $P_p = 0.85f'_cA_1$. Then

$$\phi_c(0.85f'_cA_1) \ge R_u$$
$$0.65(0.85)(3.5)A_1 \ge 99.73$$
$$A_1 \ge 51.57 \text{ in.}^2$$

The minimum value of dimension B is

$$B = \frac{A_1}{\ell_b} = \frac{51.57}{6} = 8.60 \text{ in.}$$

The flange width of a W21 × 68 is 8.27 inches, making the plate slightly wider than the flange, which is desirable. Rounding up, **try B = 10 in.**
Compute the required plate thickness:

$$n = \frac{B - 2k}{2} = \frac{10 - 2(1.19)}{2} = 3.810 \text{ in.}$$

From Equation 5.13,

$$t = \sqrt{\frac{2.22R_u n^2}{B\ell_b F_y}} = \sqrt{\frac{2.22(99.73)(3.810)^2}{10(6)(36)}} = 1.22 \text{ in.}$$

Answer Use a PL 1¼ × 6 × 10.

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ASD solution

$$w_a = w_D + w_L = 9 \text{ kips/ft}$$

$$R_a = \frac{w_a L}{2} = \frac{9(15.83)}{2} = 71.24 \text{ kips}$$

Determine the length of bearing ℓ_b required to prevent web yielding. From AISC Equation J10-3, the nominal strength is

$$R_n = F_y t_w (2.5k + \ell_b)$$

For $\dfrac{R_n}{\Omega} \geq R_a$,

$$\frac{50(0.430)[2.5(1.19) + \ell_b]}{1.50} \geq 71.24$$

$$\ell_b \geq 2.00 \text{ in.}$$

Determine the value of ℓ_b required to prevent web crippling. Assume $\ell_b/d \leq 0.2$ and use AISC Equation J10-5a:

$$\frac{R_n}{\Omega} = \frac{1}{\Omega}(0.40)t_w^2\left[1 + 3\left(\frac{\ell_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{EF_y t_f}{t_w}} \geq R_a$$

$$\frac{1}{2.00}(0.40)(0.430)^2\left[1 + 3\left(\frac{\ell_b}{21.1}\right)\left(\frac{0.430}{0.685}\right)^{1.5}\right]\sqrt{\frac{29{,}000(50)(0.685)}{0.430}} \geq 71.24$$

$$\ell_b \geq 3.78 \text{ in.}$$

$$\frac{\ell_b}{d} = \frac{3.78}{21.1} = 0.179 < 0.2 \qquad \text{(OK)}$$

Try ℓ_b = 6 in.: Conservatively assume that the full area of the support is used and determine B from a consideration of bearing strength. Using AISC Equation J8-1, we obtain

$$\frac{P_p}{\Omega_c} = \frac{0.85 f'_c A_1}{\Omega_c} \geq R_a$$

$$\frac{0.85(3.5)A_1}{2.31} \geq 71.24$$

$$A_1 \geq 55.32 \text{ in.}^2$$

The minimum value of dimension B is

$$B = \frac{A_1}{\ell_b} = \frac{55.32}{6} = 9.22 \text{ in.}$$

Try B = 10 in.:

$$n = \frac{B - 2k}{2} = \frac{10 - 2(1.19)}{2} = 3.810 \text{ in.}$$

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From Equation 5.14,

$$t \geq \sqrt{\frac{3.34R_a n^2}{B\ell_b F_y}} = \sqrt{\frac{3.34(71.24)(3.810)^2}{10(6)(36)}} = 1.27 \text{ in.}$$

Answer Use a PL 1½ × 6 × 10.

If the beam is not laterally braced at the load point (in such a way as to prevent relative lateral displacement between the loaded compression flange and the tension flange), the Specification requires that *web sidesway buckling* be investigated (AISC J10.4). When loads are applied to both flanges, *web compression buckling* must be checked (AISC J10.5).

Column Base Plates

As with beam bearing plates, the design of column base plates requires consideration of bearing pressure on the supporting material and bending of the plate. A major difference is that bending in beam bearing plates is in one direction, whereas column base plates are subjected to two-way bending. Moreover, web crippling and web yielding are not factors in column base plates design.

The background and development of the plate thickness equation is presented here in LRFD terms. After some simple modifications, the corresponding ASD equation will be given.

Column base plates can be categorized as large or small, where small plates are those whose dimensions are approximately the same as the column dimensions. Furthermore, small plates behave differently when lightly loaded than when they are more heavily loaded.

The thickness of *large* plates is determined from consideration of bending of the portions of the plate that extend beyond the column outline. Bending is assumed to take place about axes at middepth of the plate near the edges of the column flanges. Two of the axes are parallel to the web and $0.80b_f$ apart, and two axes are parallel to the flanges and $0.95d$ apart. The two 1-inch cantilever strips are labeled m and n in Figure 5.42. To determine the required plate thickness, we use Equation 5.12 with the following changes: Instead of n, use the larger of m and n from Figure 5.42 and call it ℓ. Replace ℓ_b with N and obtain

$$t \geq \sqrt{\frac{2P_u \ell^2}{0.90BNF_y}}$$

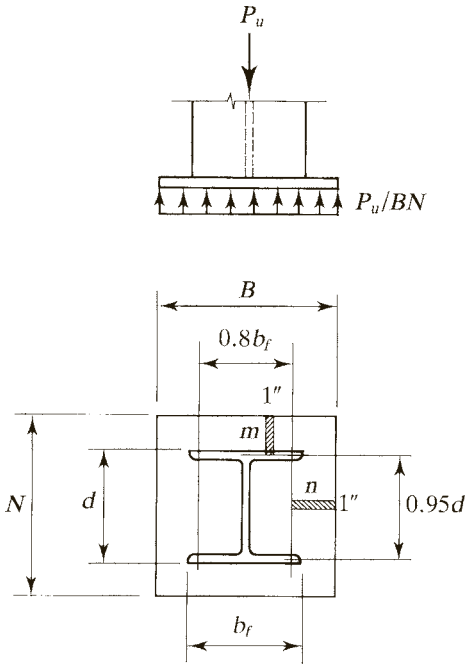
or

$$t \geq \ell \sqrt{\frac{2P_u}{0.90BNF_y}} \tag{5.15}$$

where ℓ is the larger of m and n. This approach is referred to as the *cantilever method*.

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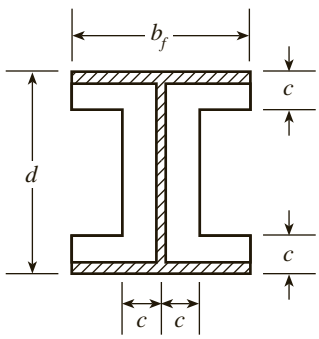
FIGURE 5.42



Lightly loaded *small* base plates can be designed by using the Murray–Stockwell method (Murray, 1983). In this approach, the portion of the column load that falls within the confines of the column cross section—that is, over an area $b_f d$—is assumed to be uniformly distributed over the H-shaped area shown in Figure 5.43. Thus the bearing pressure is concentrated near the column outline. The plate thickness is determined from a flexural analysis of a cantilever strip of unit width and of length c. This approach results in the equation

$$t \geq c\sqrt{\frac{2P_0}{0.90A_H F_y}} \tag{5.16}$$

FIGURE 5.43



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where

$$P_0 = \frac{P_u}{BN} \times b_f d$$

= load within the area $b_f d$

= load on H-shaped area

A_H = H-shaped area

c = dimension needed to give a stress of $\dfrac{P_0}{A_H}$ equal to the design bearing stress of the supporting material

Note that Equation 5.16 has the same form as Equation 5.15 but with the stress P_u/BN replaced by P_0/A_H.

For more heavily loaded base plates (the boundary between lightly loaded and heavily loaded plates is not well defined), Thornton (1990a) proposed an analysis based on two-way bending of the portion of the plate between the web and the flanges. As shown in Figure 5.44, this plate segment is assumed to be fixed at the web, simply supported at the flanges, and free at the other edge. The required thickness is

$$t \geq n' \sqrt{\frac{2P_u}{0.90BNF_y}}$$

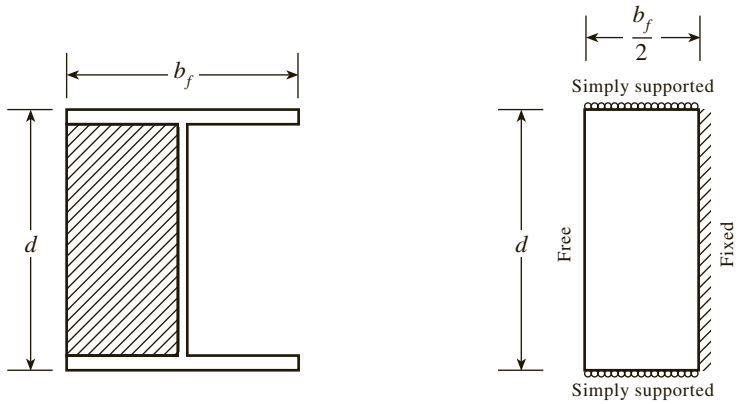
where

$$n' = \frac{1}{4}\sqrt{db_f} \tag{5.17}$$

These three approaches were combined by Thornton (1990b), and a summary of the resulting unified procedure follows. The required plate thickness is

$$t \geq \ell \sqrt{\frac{2P_u}{0.90F_yBN}} \tag{5.18}$$

FIGURE 5.44



(a) Area of Plate Considered

(b) Approximated Size and Edge Conditions

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where

$$\ell = \max(m, n, \lambda n')$$

$$m = \frac{N - 0.95d}{2}$$

$$n = \frac{B - 0.8b_f}{2}$$

$$\lambda = \frac{2\sqrt{X}}{1 + \sqrt{1 - X}} \leq 1$$

$$X = \left(\frac{4db_f}{(d + b_f)^2}\right)\frac{P_u}{\phi_c P_p}$$

$$n' = \frac{1}{4}\sqrt{db_f}$$

$$\phi_c = 0.65$$

P_p = nominal bearing strength from AISC Equation J8-1 or J8-2

There is no need to determine whether the plate is large or small, lightly loaded, or heavily loaded. As a simplification, λ can always be conservatively taken as 1.0 (Thornton, 1990b).

This procedure is the same as that given in Part 14 of the *Manual*, "Design of Beam Bearing Plates, Column Base Plates, Anchor Rods, and Column Splices."

For ASD, we rewrite Equation 5.18 by substituting P_a for P_u and $1/\Omega$ for ϕ:

$$t \geq \ell\sqrt{\frac{2P_a}{(1/\Omega)F_y BN}}$$

or

$$t \geq \ell\sqrt{\frac{2P_a}{F_y BN/1.67}} \tag{5.19}$$

In the equation for X, we again substitute P_a for P_u and $1/\Omega$ for ϕ:

$$X = \left(\frac{4db_f}{(d + b_f)^2}\right)\frac{P_a}{P_p/\Omega_c} \tag{5.20}$$

where $\Omega_c = 2.31$.

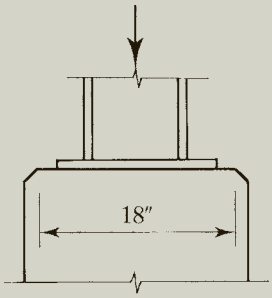
The equations for ℓ, m, n, λ, and n' are the same as for LRFD.

Example 5.19

A W10 × 49 is used as a column and is supported by a concrete pier as shown in Figure 5.45. The top surface of the pier is 18 inches by 18 inches. Design an A36 base plate for a column dead load of 98 kips and a live load of 145 kips. The concrete strength is $f'_c = 3000$ psi.

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FIGURE 5.45



LRFD solution

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(98) + 1.6(145) = 349.6 \text{ kips}$$

Compute the required bearing area. From AISC Equation J8-2,

$$P_p = 0.85f'_c A_1 \sqrt{\frac{A_2}{A_1}} \leq 1.7f'_c A_1$$

For $\phi_c P_p \geq P_u$,

$$0.65\left[0.85(3)A_1\sqrt{\frac{18(18)}{A_1}}\right] \geq 349.6$$

$$A_1 \geq 137.3 \text{ in.}^2$$

Check upper limit:

$$\phi_c 1.7f'_c A_1 = 0.65(1.7)(3)(137.3) = 455 \text{ kips} > 349.6 \text{ kips} \qquad \text{(OK)}$$

Also, the plate must be at least as large as the column, so

$$b_f d = 10.0(10.0) = 100 \text{ in.}^2 < 137.3 \text{ in.}^2 \qquad \text{(OK)}$$

For $B = N = 13$ in., A_1 provided $= 13(13) = 169$ in.^{2}

The dimensions of the cantilever strips are

$$m = \frac{N - 0.95d}{2} = \frac{13 - 0.95(10)}{2} = 1.75 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{13 - 0.8(10)}{2} = 2.5 \text{ in.}$$

From Equation 5.17,

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{10.0(10.0)} = 2.5 \text{ in.}$$

As a conservative simplification, let $\lambda = 1.0$, giving

$$\ell = \max(m, n, \lambda n') = \max(1.75, 2.5, 2.5) = 2.5 \text{ in.}$$

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From Equation 5.18, the required plate thickness is

$$t = \ell\sqrt{\frac{2P_u}{0.90F_yBN}} = 2.5\sqrt{\frac{2(349.6)}{0.90(36)(13)(13)}} = 0.893 \text{ in.}$$

Answer Use a PL $1 \times 13 \times 13$.

ASD solution The applied load is $P_a = D + L = 98 + 145 = 243$ kips.

Compute the required bearing area. Using AISC Equation J8-2,

$$\frac{P_p}{\Omega_c} \geq P_a$$

$$\frac{1}{2.31}\left[0.85(3)A_1\sqrt{\frac{18(18)}{A_1}}\right] \geq 243$$

$$A_1 \geq 150 \text{ in.}^2$$

The upper limit is

$$\frac{1}{\Omega_c}(1.7f_c'A_1) = \frac{1}{2.31}[(1.7)(3)(150)] = 331 \text{ kips} > 243 \text{ kips} \qquad \text{(OK)}$$

The plate must be as large as the column:

$$b_f d = 10.0(10.0) = 100 \text{ in.}^2 < 150 \text{ in.}^2 \quad \text{(OK)}$$

Try $B = N = 13$ in., with A_1 provided $= 13(13) = 169 \text{ in.}^2 > 150 \text{ in.}^2$

$$m = \frac{N - 0.95d}{2} = \frac{13 - 0.95(10.0)}{2} = 1.75 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{13 - 0.8(10)}{2} = 2.5 \text{ in.}$$

$$n' = \frac{1}{4}\sqrt{db_f} = \frac{1}{4}\sqrt{10.0(10.0)} = 2.5 \text{ in.}$$

Conservatively, let $\lambda = 1.0$, resulting in

$$\ell = \max(m, n, \lambda n') = \max(1.75, 2.5, 2.5) = 2.5 \text{ in.}$$

From Equation 5.19,

$$t \geq \ell\sqrt{\frac{2P_a}{F_yBN/1.67}} = 2.5\sqrt{\frac{2(243)}{36(13)(13)/1.67}} = 0.913 \text{ in.}$$

Answer Use a PL $1 \times 13 \times 13$.

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5.15 BIAXIAL BENDING

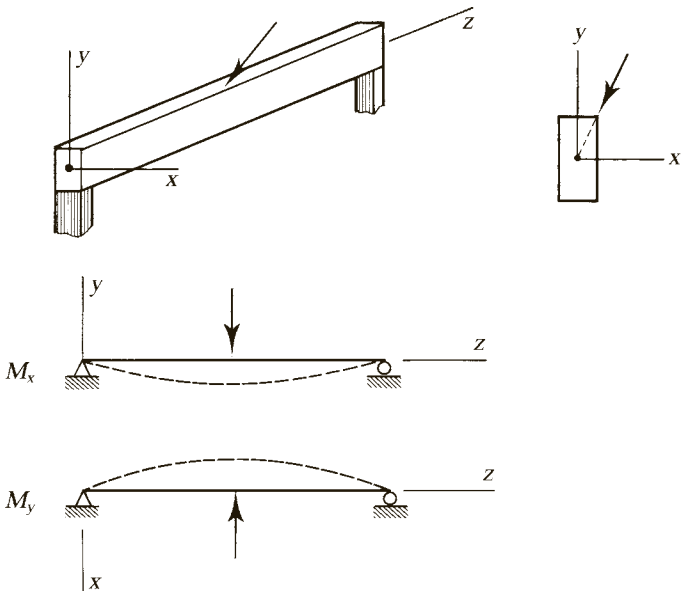
Biaxial bending occurs when a beam is subjected to a loading condition that produces bending about both the major (strong) axis and the minor (weak) axis. Such a case is illustrated in Figure 5.46, where a single concentrated load acts normal to the longitudinal axis of the beam but is inclined with respect to each of the principal axes of the cross section. Although this loading is more general than those previously considered, it is still a special case: The load passes through the shear center of the cross section. The shear center is that point through which the loads must act if there is to be no twisting, or torsion, of the beam. The location of the shear center can be determined from elementary mechanics of materials by equating the internal resisting torsional moment, derived from the shear flow on the cross section, to the external torque.

The location of the shear center for several common cross sections is shown in Figure 5.47a, where the shear center is indicated by a circle. The value of e_o, which locates the shear center for channel shapes, is tabulated in the *Manual*. The shear center is always located on an axis of symmetry; thus the shear center will be at the centroid of a cross section with two axes of symmetry. Figure 5.47b shows the deflected position of two different beams when loads are applied through the shear center and when they are not.

Case I: Loads Applied Through the Shear Center

If loads act through the shear center, the problem is one of simple bending in two perpendicular directions. As illustrated in Figure 5.48, the load can be resolved into rectangular components in the x- and y-directions, each producing bending about a different axis.

FIGURE 5.46



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FIGURE 5.47

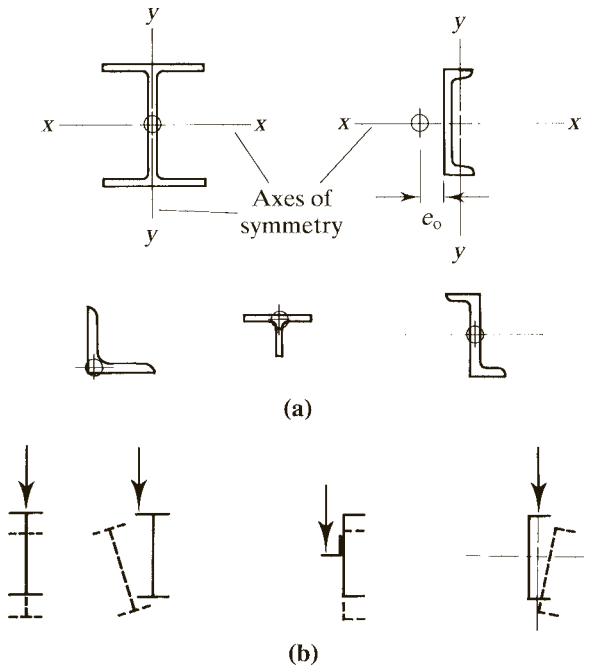
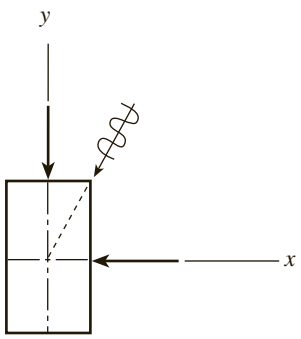


FIGURE 5.48



To deal with this combined loading, we temporarily look ahead to Chapter H of the Specification, "Design of Members for Combined Forces and Torsion" (also the subject of Chapter 6 of this textbook, "Beam–Columns"). The Specification deals with combined loading primarily through the use of interaction formulas, which account for the relative importance of each load effect in relation to the strength corresponding to that effect. For example, if there is bending about one axis only, we can write for that axis

$$\text{Required moment strength} \leq \text{available moment strength}$$

or

$$\frac{\text{Required moment strength}}{\text{Available moment strength}} \leq 1.0$$

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If there is bending about both the x and y axes, the interaction approach requires that the sum of ratios for the two effects be less than 1.0; that is,

$$\frac{\text{Required } x\text{-axis moment strength}}{\text{Available } x\text{-axis moment strength}} + \frac{\text{required } y\text{-axis moment strength}}{\text{available } y\text{-axis moment strength}} \leq 1.0 \quad (5.21)$$

In effect, this approach allows the designer to allocate to one direction what has not been "used up" in the other direction. AISC Section H1 incorporates a comparable ratio for axial loads and gives two interaction formulas, one for small axial loads and one for large axial loads (we discuss the reason for this in Chapter 6). With biaxial bending and no axial load, the formula for small axial load is the appropriate one, and it reduces to the form of Equation 5.21. (The exact notation and form of the AISC Equations are covered in Chapter 6.)

For LRFD, Equation 5.21 becomes

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \leq 1.0 \quad (5.22)$$

where

M_{ux} = factored-load moment about the x axis
M_{nx} = nominal moment strength for x-axis bending
M_{uy} = factored-load moment about the y axis
M_{ny} = nominal moment strength for the y axis

For ASD,

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} \leq 1.0 \quad (5.23)$$

where

M_{ax} = service load moment about the x axis
M_{ay} = service load moment about the y axis

Weak-Axis Bending Strength

To this point, the strength of I-shaped cross sections bent about the weak axis has not been considered. Doing so is relatively simple. Any shape bent about its weak axis cannot buckle in the other direction, so lateral-torsional buckling is not a limit state. If the shape is compact, then

$$M_{ny} = M_{py} = F_y Z_y \leq 1.6 F_y S_y \quad \text{(AISC Equation F6-1)}$$

where

M_{ny} = nominal moment strength about the y axis
M_{py} = plastic moment strength about the y axis
$F_y S_y$ = yield moment for the y axis

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(The y subscripts in M_{ny} and M_{py} are not in the Specification; they have been added here.) The limit of $1.6F_yS_y$ is to prevent excessive working load deformation and is satisfied when

$$\frac{Z_y}{S_y} \leq 1.6$$

If the shape is noncompact because of the flange width-to-thickness ratio, the strength will be given by

$$M_{ny} = M_{py} - (M_{py} - 0.7F_yS_y)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{(AISC Equation F6-2)}$$

This is the same as AISC Equation F3-1 for flange local buckling, except for the axis of bending.

Example 5.20

A W21 × 68 is used as a simply supported beam with a span length of 12 feet. Lateral support of the compression flange is provided only at the ends. Loads act through the shear center, producing moments about the x and y axes. The service load moments about the x axis are $M_{Dx} = 48$ ft-kips and $M_{Lx} = 144$ ft-kips. Service load moments about the y axis are $M_{Dy} = 6$ ft-kips and $M_{Ly} = 18$ ft-kips. If A992 steel is used, does this beam satisfy the provisions of the AISC Specification? Assume that all moments are uniform over the length of the beam.

Solution First, compute the nominal flexural strength for x-axis bending. The following data for a W21 × 68 are obtained from the Z_x table. The shape is compact (no footnote to indicate otherwise) and

$$L_p = 6.36 \text{ ft}, L_r = 18.7 \text{ ft}$$

The unbraced length $L_b = 12$ ft, so $L_p < L_b < L_r$, and the controlling limit state is inelastic lateral-torsional buckling. Then

$$M_{nx} = C_b\left[M_{px} - (M_{px} - 0.7F_yS_x\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_{px}$$

$$M_{px} = F_yZ_x = 50(160) = 8000 \text{ in.-kips}$$

Because the bending moment is uniform, $C_b = 1.0$.

$$M_{nx} = 1.0\left[8000 - (8000 - 0.7 \times 50 \times 140)\left(\frac{12 - 6.36}{18.7 - 6.36}\right)\right]$$

$$= 6583 \text{ in.-kips} = 548.6 \text{ ft-kips}$$

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For the *y* axis, since the shape is compact, there is no flange local buckling and

$$M_{ny} = M_{py} = F_y Z_y = 50(24.4) = 1220 \text{ in.-kips} = 101.7 \text{ ft-kips}$$

Check the upper limit:

$$\frac{Z_y}{S_y} = \frac{24.4}{15.7} = 1.55 < 1.6 \qquad \therefore\ M_{ny} = M_{py} = 101.7 \text{ in.-kips}$$

LRFD solution

For *x*-axis bending,

$$M_{ux} = 1.2M_{Dx} + 1.6M_{Lx} = 1.2(48) + 1.6(144) = 288.0 \text{ ft-kips}$$

For *y*-axis bending,

$$M_{uy} = 1.2M_{Dy} + 1.6M_{Ly} = 1.2(6) + 1.6(18) = 36.0 \text{ ft-kips}$$

Check interaction Equation 5.22:

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{288.0}{0.90(548.6)} + \frac{36.0}{0.90(101.7)} = 0.977 < 1.0 \qquad \text{(OK)}$$

(Note that $\phi_b M_{nx}$ can be obtained from the beam design charts or *Manual* Table 6-2.)

Answer The W21 × 68 is satisfactory.

ASD solution

For *x*-axis bending,

$$M_{ax} = M_{Dx} + M_{Lx} = 48 + 144 = 192 \text{ ft-kips}$$

For *y*-axis bending,

$$M_{ay} = M_{Dy} + M_{Ly} = 6 + 18 = 24 \text{ ft-kips}$$

Check interaction Equation 5.23:

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{192}{548.6/1.67} + \frac{24}{101.7/1.67} = 0.979 < 1.0 \quad \text{(OK)}$$

(Note that M_{nx}/Ω_b can be obtained from the beam design charts or *Manual* Table 6-2.)

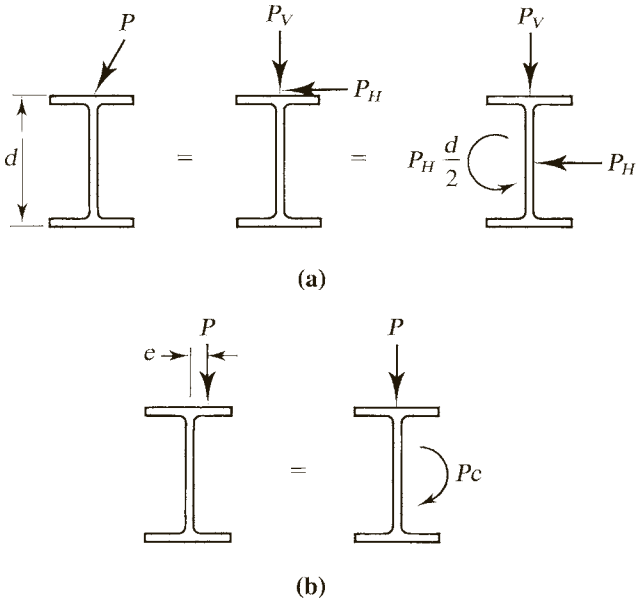
Answer The W21 × 68 is satisfactory.

Case II: Loads Not Applied Through the Shear Center

When loads are not applied through the shear center of a cross section, the result is flexure plus torsion. If possible, the structure or connection geometry should be modified to remove the eccentricity. The problem of torsion in rolled shapes is a complex one, and we resort to approximate, although conservative, methods for dealing with it.

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FIGURE 5.49



A more detailed treatment of this topic, complete with design aids, can be found in *Torsional Analysis of Structural Steel Members* (AISC, 1997b). A typical loading condition that gives rise to torsion is shown in Figure 5.49a. The resultant load is applied to the center of the top flange, but its line of action does not pass through the shear center of the section. As far as equilibrium is concerned, the force can be moved to the shear center provided that a couple is added. The equivalent system thus obtained will consist of the given force acting through the shear center plus a twisting moment, as shown. In Figure 5.49b, there is only one component of load to contend with, but the concept is the same.

Figure 5.50 illustrates a simplified way of treating these two cases. In Figure 5.50a, the top flange is assumed to provide the total resistance to the horizontal component of the load. In Figure 5.50b, the twisting moment Pe is resisted by a couple consisting of equal forces acting at each flange. As an approximation, each flange can be considered to resist each of these forces independently. Consequently, the problem is reduced to a case of bending of two shapes, each one loaded through its shear center. In each of the two situations depicted in Figure 5.50, only about half the cross section is considered to be effective with respect to its y axis; therefore, when considering the strength of a single flange, use half the tabulated value of Z_y for the shape.

Design of Roof Purlins

Roof purlins that are part of a sloping roof system can be subjected to biaxial bending of the type just described. For the roof purlin shown in Figure 5.51, the load is vertical, but the axes of bending are inclined. This condition corresponds to the loading of Figure 5.50a. The component of load normal to the roof will cause bending about the

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FIGURE 5.50

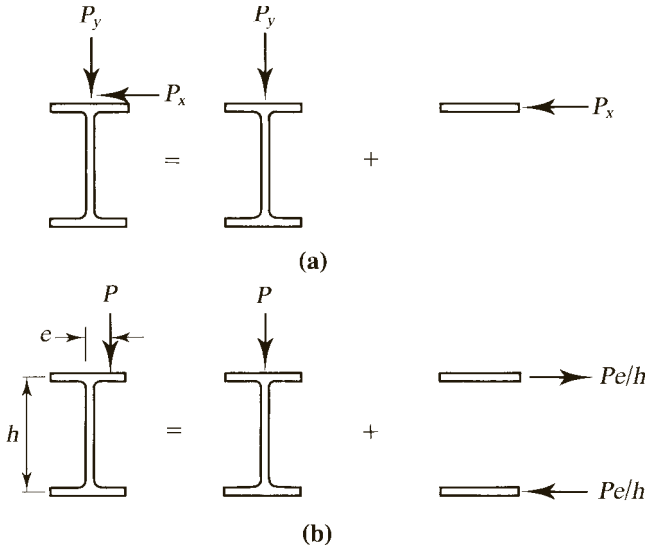
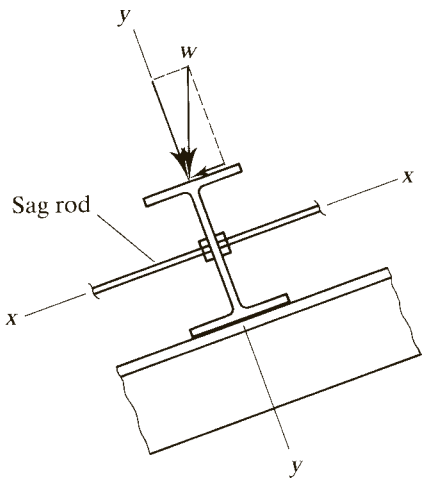


FIGURE 5.51



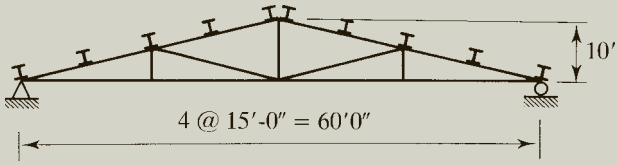
x axis, and the parallel component bends the beam about its y axis. If the purlins are simply supported at the trusses (or rigid frame rafters), the maximum bending moment about each axis is $wL^2/8$, where w is the appropriate component of load. If sag rods are used, they will provide lateral support with respect to x-axis bending and will act as transverse supports for y-axis bending, requiring that the purlin be treated as a continuous beam. For uniform sag-rod spacings, the formulas for continuous beams in Part 3 of the *Manual* can be used.

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Example 5.21

A roof system consists of trusses of the type shown in Figure 5.52 spaced 15 feet apart. Purlins are to be placed at the joints and at the midpoint of each top-chord member. Sag rods will be located at the center of each purlin. The total gravity load, including an estimated purlin weight, is 42 psf of roof surface, with a live-load–to–dead-load ratio of 1.0. Assuming that this is the critical loading condition, use A36 steel and select a channel shape for the purlins.

FIGURE 5.52



LRFD solution

For the given loading condition, dead load plus a roof live load with no wind or snow, load combination 3 will control:

$$w_u = 1.2w_D + 1.6L_r = 1.2(21) + 1.6(21) = 58.80 \text{ psf}$$

The width of roof surface tributary to each purlin is

$$\frac{15}{2}\frac{\sqrt{10}}{3} = 7.906 \text{ ft}$$

Then

$$\text{Purlin load} = 58.80(7.906) = 464.9 \text{ lb/ft}$$

$$\text{Normal component} = \frac{3}{\sqrt{10}}(464.9) = 441.0 \text{ lb/ft}$$

$$\text{Parallel component} = \frac{1}{\sqrt{10}}(464.9) = 147.0 \text{ lb/ft}$$

and

$$M_{ux} = \frac{1}{8}(0.4410)(15)^2 = 12.40 \text{ ft-kips}$$

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 3-22c, "Continuous Beams," the maximum moment in a two-span continuous beam with equal spans is at the interior support and is given by

$$M = 0.125w\ell^2$$

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where

w = uniform load intensity
ℓ = span length

The maximum moment about the y axis is therefore

$$M_{uy} = 0.125(0.1470)(15/2)^2 = 1.034 \text{ ft-kips}$$

To select a trial shape, use the beam design charts and choose a shape with a relatively large margin of strength with respect to major axis bending. For an unbraced length of $^{15}/_{2} = 7.5$ ft, **try a C10 × 15.3**.

For $C_b = 1.0$, $\phi_b M_{nx} = 33.0$ ft-kips. From Figure 5.16, C_b is 1.30 for the load and lateral support conditions of this beam. Therefore,

$$\phi_b M_{nx} = 1.30(33.0) = 42.90 \text{ ft-kips}$$

From the uniform load table for C shapes,

$$\phi_b M_{px} = 42.9 \text{ ft-kips} \quad \therefore \text{ Use } \phi_b M_{nx} = 42.9 \text{ ft-kips.}$$

This shape is compact (no footnote in the uniform load tables), so

$$\phi_b M_{ny} = \phi_b M_{py} = \phi_b F_y Z_y = 0.90(36)(2.34) = 75.82 \text{ in.-kips} = 6.318 \text{ ft-kips}$$

But

$$\frac{Z_y}{S_y} = \frac{2.34}{1.15} = 2.03 > 1.6$$

$$\therefore \ \phi_b M_{ny} = \phi_b(1.6F_y S_y) = 0.90(1.6)(36)(1.15)$$
$$= 59.62 \text{ in.-kips} = 4.968 \text{ ft-kips}$$

Because the load is applied to the top flange, use only half this capacity to account for the torsional effects. From Equation 5.22,

$$\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} = \frac{12.40}{42.9} + \frac{1.034}{4.968/2} = 0.705 < 1.0 \qquad \text{(OK)}$$

The shear is

$$V_u = \frac{0.4410(15)}{2} = 3.31 \text{ kips}$$

From the uniform load tables,

$$\phi_v V_n = 46.7 \text{ kips} > 3.31 \text{ kips} \qquad \text{(OK)}$$

Answer Use a C10 × 15.3.

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ASD solution

For dead load plus a roof live load, load combination 3 will control:

$$q_a = q_D + q_{Lr} = 42 \text{ psf}$$

The width of roof surface tributary to each purlin is

$$\frac{15}{2}\frac{\sqrt{10}}{3} = 7.906 \text{ ft}$$

Then

$$\text{Purlin load} = 42(7.906) = 332.1 \text{ lb/ft}$$

$$\text{Normal component} = \frac{3}{\sqrt{10}}(332.1) = 315.1 \text{ lb/ft}$$

$$\text{Parallel component} = \frac{1}{\sqrt{10}}(332.1) = 105.0 \text{ lb/ft}$$

and

$$M_{ax} = \frac{1}{8}wL^2 = \frac{1}{8}(0.3151)(15)^2 = 8.862 \text{ ft-kips}$$

With sag rods placed at the midpoint of each purlin, the purlins are two-span continuous beams with respect to weak axis bending. From Table 3-22c, "Continuous Beams", the maximum moment in a two-span continuous beam with equal spans is at the interior support and is given by

$$M = 0.125w\ell^2$$

where

w = uniform load intensity

ℓ = span length

The maximum moment about the y axis is therefore

$$M_{ay} = 0.125(0.1050)(15/2)^2 = 0.7383 \text{ ft-kips}$$

To select a trial shape, use the beam design charts and choose a shape with a relatively large margin of strength with respect to major axis bending. For an unbraced length of 15/2 = 7.5 ft, try a **C10 × 15.3**.

For $C_b = 1.0$, $M_{nx}/\Omega_b = 22.0$ ft-kips. From Figure 5.16, $C_b = 1.30$ for the load and lateral support conditions of this beam. Therefore,

$$M_{nx}/\Omega_b = 1.30(22.0) = 28.60 \text{ ft-kips}$$

From the uniform load tables,

$$M_{px}/\Omega_b = 28.6 \text{ ft-kips}$$

$\therefore$ Use $M_{nx}/\Omega_b = 28.6$ ft-kips.

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This shape is compact (no footnote in the uniform load tables), so

$$M_{ny}/\Omega_b = M_{py}/\Omega_b = F_y Z_y/\Omega_b = 36(2.34)/1.67 = 50.44 \text{ in.-kips} = 4.203 \text{ ft-kips}$$

But

$$\frac{Z_y}{S_y} = \frac{2.34}{1.15} = 2.03 > 1.6$$

$$\therefore M_{ny}/\Omega_b = 1.6F_y S_y/\Omega_b = 1.6(36)(1.15)/1.67 = 39.66 \text{ in.-kips} = 3.300 \text{ ft-kips.}$$

Because the load is applied to the top flange, use only half of this capacity to account for the torsional effects. From Equation 5.23,

$$\frac{M_{ax}}{M_{nx}/\Omega_b} + \frac{M_{ay}}{M_{ny}/\Omega_b} = \frac{8.862}{28.6} + \frac{0.7383}{3.300/2} = 0.757 < 1.0 \qquad \text{(OK)}$$

The maximum shear is

$$V_a = \frac{0.3151(15)}{2} = 2.36 \text{ kips}$$

From the uniform load tables,

$$\frac{V_n}{\Omega_v} = 31.0 \text{ kips} > 2.36 \text{ kips} \qquad \text{(OK)}$$

Answer Use a C10 × 15.3.

5.16 BENDING STRENGTH OF VARIOUS SHAPES

W, S, M, and C shapes are the most commonly used hot-rolled shapes for beams, and their bending strength has been covered in the preceding sections. Other shapes are sometimes used as flexural members, however, and this section provides a summary of some of the relevant AISC provisions. All equations are from Chapter F of the Specification. (The width-to-thickness limits are from Chapter B.) No numerical examples are given in this section, but Example 6.12 includes the computation of the flexural strength of a structural tee-shape.

1. **Square and Rectangular HSS (AISC F7):** (Although AISC F7 includes box-shaped members, we do not cover them here; all provisions herein apply to HSS only.)
   a. **Width-to-thickness parameters** (see Figure 5.53):
      i. Flange:

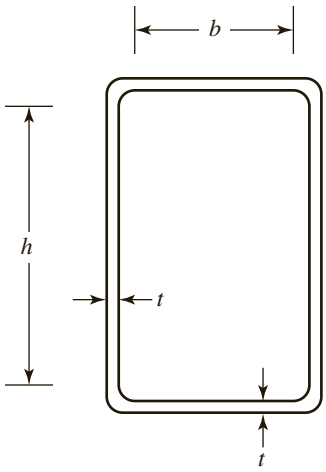
$$\lambda = \frac{b}{t} \qquad \lambda_p = 1.12\sqrt{\frac{E}{F_y}} \qquad \lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$

      ii. Web:

$$\lambda = \frac{h}{t} \qquad \lambda_p = 2.42\sqrt{\frac{E}{F_y}} \qquad \lambda_r = 5.70\sqrt{\frac{E}{F_y}}$$

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FIGURE 5.53



If the actual dimensions b and h are not known, they may be estimated as the total width or depth minus three times the thickness. The *design* thickness, which is 0.93 times the *nominal* thickness, should be used. (b/t and h/t ratios for HSS are given in the *Manual* in Part 1, "Dimensions and Properties.")

b. **Bending about either axis (loaded in the plane of symmetry):** The nominal strength is the smallest of the limit states of yielding, flange local buckling, and web local buckling. (Because of the high torsional resistance of closed cross-sectional shapes, lateral-torsional buckling of HSS need not be considered, even for rectangular shapes bent about the strong axis. This is discussed in the Commentary to AISC F7.)

i. Yielding:

$$M_n = M_p = F_y Z \qquad \text{(AISC Equation F7-1)}$$

ii. Flange Local Buckling:

$$M_n = M_p - (M_p - F_y S)\left(3.57\frac{b}{t_f}\sqrt{\frac{F_y}{E}} - 4.0\right) \le M_p$$

(AISC Equation F7-2)

iii. Web Local Buckling:

$$M_n = M_p - (M_p - F_y S)\left(3.05\frac{h}{t_w}\sqrt{\frac{F_y}{E}} - 0.738\right) \le M_p$$

(AISC Equation F7-6)

2. **Round HSS (AISC F8):**

a. **Width-to-thickness parameters:**

$$\lambda = \frac{D}{t} \qquad \lambda_p = \frac{0.07E}{F_y} \qquad \lambda_r = \frac{0.31E}{F_y}$$

where D is the outer diameter.

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b. **Nominal bending strength:** There is no LTB limit state for circular (or square) shapes.

i. Compact shapes:

$$M_n = M_p = F_y Z \qquad \text{(AISC Equation F8-1)}$$

ii. Noncompact shapes:

$$M_n = \left(\frac{0.021E}{D/t} + F_y\right)S \qquad \text{(AISC Equation F8-2)}$$

iii. Slender shapes:

$$M_n = F_{cr} S \qquad \text{(AISC Equation F8-3)}$$

where

$$F_{cr} = \frac{0.33E}{D/t} \qquad \text{(AISC Equation F8-4)}$$

3. **Tees and Double Angles Loaded in the Plane of Symmetry:** Tees and double-angle shapes are frequently used as truss members, and these members are sometimes subject to bending as well as axial compression, making them beam-columns (Chapter 6 of this book).

The flexural strength is the smallest value obtained from the limit states of yielding, lateral-torsional buckling, flange local buckling, and local buckling of tee stems and double-angle web legs.

a. **Yielding:**

$$M_n = M_p \qquad \text{(AISC Equation F9-1)}$$

i. For tee stems and double-angle web legs in tension,

$$M_p = F_y Z_x \le 1.6M_y \qquad \text{(AISC Equation F9-2)}$$

$$M_y = F_y S_x \qquad \text{(AISC Equation F9-3)}$$

ii. For tee stems in compression,

$$M_p = M_y \qquad \text{(AISC Equation F9-4)}$$

iii. For double angles with web legs in compression,

$$M_p = 1.5M_y \qquad \text{(AISC Equation F9-5)}$$

b. **Lateral-torsional buckling:**

i. For tee stems and double-angle web legs in tension, when $L_p < L_b \le L_r$,

$$M_n = M_p - (M_p - M_y)\left(\frac{L_b - L_p}{L_r - L_p}\right) \qquad \text{(AISC Equation F9-6)}$$

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when $L_b > L_r$,

$$M_n = M_{cr} \quad \text{(AISC Equation F9-7)}$$

where

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \quad \text{(AISC Equation F9-8)}$$

$$L_r = 1.95 \left(\frac{E}{F_y}\right) \frac{\sqrt{I_y J}}{S_x} \sqrt{2.36 \left(\frac{F_y}{E}\right) \frac{d S_x}{J} + 1} \quad \text{(AISC Equation F9-9)}$$

$$M_{cr} = \frac{1.95E}{L_b} \sqrt{I_y J} (B + \sqrt{1 + B^2}) \quad \text{(AISC Equation F9-10)}$$

$$B = 2.3 \left(\frac{d}{L_b}\right) \sqrt{\frac{I_y}{J}} \quad \text{(AISC Equation F9-11)}$$

d = depth of tee

ii. For stems and double-angle web legs in compression anywhere along the unbraced length,

$$M_n = M_{cr} \leq M_y \quad \text{(AISC Equation F9-13)}$$

where

$$M_{cr} = \frac{1.95E}{L_b} \sqrt{I_y J} (B + \sqrt{1 + B^2}) \quad \text{(AISC Equation F9-10)}$$

and

$$B = -2.3 \left(\frac{d}{L_b}\right) \sqrt{\frac{I_y}{J}} \quad \text{(AISC Equation F9-12)}$$

(This is the same as AISC Equation F9-11 except for the negative sign.)

iii. For double-angle web legs,

when $\dfrac{M_y}{M_{cr}} \leq 1.0$,

$$M_n = \left(1.92 - 1.17 \sqrt{\frac{M_y}{M_{cr}}}\right) M_y \leq 1.5 M_y \quad \text{(AISC Equation F10-2)}$$

when $\dfrac{M_y}{M_{cr}} > 1.0$,

$$M_n = \left(0.92 - \frac{0.17 M_{cr}}{M_y}\right) M_{cr} \quad \text{(AISC Equation F10-3)}$$

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c. **Flange local buckling:**

i. For noncompact tee flanges,

$$M_n = M_p - (M_p - 0.7F_yS_{xc})\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right) \le 1.6M_y$$

(AISC Equation F9-14)

ii. For slender tee flanges,

$$M_n = \frac{0.7ES_{xc}}{(b_f/2t_f)^2}$$

(AISC Equation F9-15)

iii. For noncompact double-angle flange legs,

$$M_n = F_yS_c\left[2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right]$$

(AISC Equation F10-6)

iv. For slender double-angle flange legs,

$$M_n = F_{cr}S_c$$

(AISC Equation F10-7)

where

$$F_{cr} = \frac{0.71E}{(b/t)^2}$$

(AISC Equation F10-8)

b = full width of leg in compression

S_c = elastic section modulus referred to toe of the leg in compression

d. **Local buckling of tee stems and double-angle web legs:**

i. For tee stems,

$$M_n = F_{cr}S_x$$

(AISC Equation F9-16)

Compact webs: $F_{cr} = F_y$ (AISC Equation F9-17)

Noncompact webs: $F_{cr} = \left(1.43 - 0.515\,\frac{d}{t_w}\sqrt{\frac{F_y}{E}}\right)F_y$

(AISC Equation F9-18)

Slender webs: $F_{cr} = \frac{1.52E}{(d/t_w)^2}$ (AISC Equation F9-19)

ii. For double-angle web legs,

Noncompact legs: $M_n = F_yS_c\left[2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right]$

(AISC Equation F10-6)

Slender legs: $M_n = F_{cr}S_c$ (AISC Equation F10-7)

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where

$$F_{cr} = \frac{0.71E}{(b/t)^2} \qquad \text{(AISC Equation F10-8)}$$

b = full width of leg in compression

S_c = elastic section modulus referred to toe of the leg in compression

4. **Solid Rectangular Bars (AISC F11):** The applicable limit states are yielding and LTB for major axis bending; local buckling is not a limit state for either major or minor axis bending.
   a. **Bending about the major axis:**

For $\dfrac{L_b d}{t^2} \le \dfrac{0.08E}{F_y}$,

$$M_n = M_p = F_y Z \le 1.6M_y \qquad \text{(AISC Equation F11-1)}$$

where M_y = yield moment = $F_y S_x$.

For $\dfrac{0.08E}{F_y} < \dfrac{L_b d}{t^2} \le \dfrac{1.9E}{F_y}$,

$$M_n = C_b\left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right]M_y \le M_p \qquad \text{(AISC Equation F11-2)}$$

For $\dfrac{L_b d}{t^2} > \dfrac{1.9E}{F_y}$,

$$M_n = F_{cr} S_x \le M_p \qquad \text{(AISC Equation F11-3)}$$

where

$$F_{cr} = \frac{1.9EC_b}{L_b d/t^2} \qquad \text{(AISC Equation F11-4)}$$

t = width of bar (dimension parallel to axis of bending)

d = depth of bar

   b. **Bending about the minor axis:**

$$M_n = M_p = F_y Z \le 1.6M_y \qquad \text{(AISC Equation F11-1)}$$

where M_y = yield moment = $F_y S_x$.

5. **Solid Circular Bars (AISC F11):**

$$M_n = M_p = F_y Z \le 1.6M_y \qquad \text{(AISC Equation F11-1)}$$

(For a circle, $Z/S = 1.7 > 1.6$, so the upper limit *always* controls.)

For flexural members not covered in this summary (single angles, other doubly unsymmetrical shapes, and shapes built up from plate elements), refer to Chapter F of the AISC Specification. (Shapes built up from plate elements are also covered in Chapter 10 of this book.)

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PROBLEMS

Bending Stress and the Plastic Moment

5.2-1 A flexural member is fabricated from two flange plates ½ × 7½ and a web plate ⅜ × 17. The yield stress of the steel is 50 ksi.

a. Compute the plastic section modulus Z and the plastic moment M_p with respect to the major principal axis.

b. Compute the elastic section modulus S and the yield moment M_y with respect to the major principal axis.

5.2-2 An unsymmetrical flexural member consists of a ½ × 12 top flange, a ½ × 7 bottom flange, and a ⅜ × 16 web.

a. Determine the distance $\bar{y}$ from the top of the shape to the horizontal *plastic* neutral axis.

b. If A572 Grade 50 steel is used, what is the plastic moment M_p for the horizontal plastic neutral axis?

c. Compute the plastic section modulus Z with respect to the minor principal axis.

5.2-3 Verify the value of Z_x for a W18 × 50 that is tabulated in the dimensions and properties tables in Part 1 of the *Manual*.

5.2-4 Verify the value of Z_x given in the *Manual* for an S10 × 35.

Classification of Shapes

5.4-1 For W-, M-, and S-shapes with $F_y = 60$ ksi:

a. List the shapes in Part 1 of the *Manual* that are noncompact (when used as flexural members). State whether they are noncompact because of the flange, the web, or both.

b. List the shapes in Part 1 of the *Manual* that are slender. State whether they are slender because of the flange, the web, or both.

5.4-2 Repeat Problem 5.4-1 for steel with $F_y = 65$ ksi.

5.4-3 Determine the smallest value of yield stress F_y for which a W-, M-, or S-shape from Part 1 of the *Manual* will become slender. To which shapes does this value apply? What conclusion can you draw from your answer?

Bending Strength of Compact Shapes

5.5-1 The beam shown in Figure P5.5-1 is a W10 × 77 and has continuous lateral support. The load P is a service live load. If $F_y = 50$ ksi, what is the maximum permissible value of P?

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a. Use LRFD.

b. Use ASD.

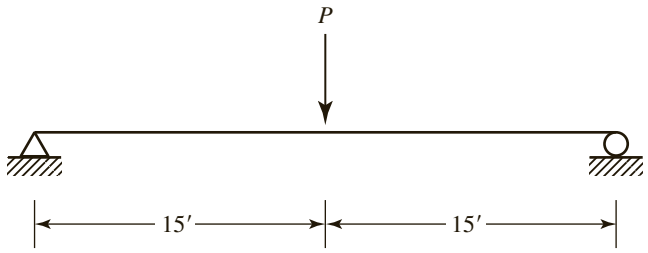


FIGURE P5.5-1

5.5-2 The given beam has continuous lateral support. If the live load is twice the dead load, what is the maximum total *service* load, in kips/ft, that can be supported? A992 steel is used.

a. Use LRFD.

b. Use ASD.

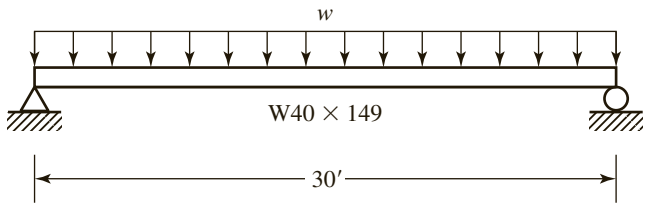


FIGURE P5.5-2

5.5-3 A simply supported beam is subjected to a uniform service dead load of 1.0 kips/ft (including the weight of the beam), a uniform service live load of 2.0 kips/ft, and a concentrated service dead load of 40 kips. The beam is 40 feet long, and the concentrated load is located 15 feet from the left end. The beam has continuous lateral support, and A572 Grade 50 steel is used. Is a W30 × 108 adequate?

a. Use LRFD.

b. Use ASD.

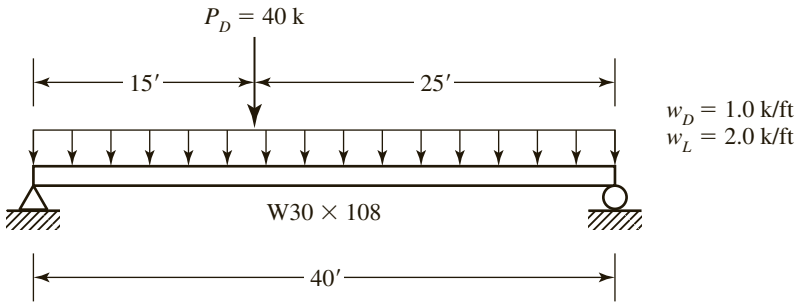


FIGURE P5.5-3

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5.5-4 The beam shown in Figure P5.5-4 has continuous lateral support of both flanges. The uniform load is a service load consisting of 50% dead load and 50% live load. The dead load includes the weight of the beam. If A992 steel is used, is a W12 × 35 adequate?

a. Use LRFD.

b. Use ASD.

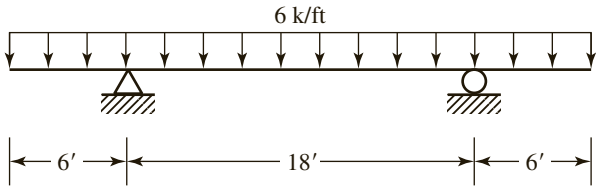


FIGURE P5.5-4

5.5-5 The beam shown in Figure P5.5-5 is a two-span beam with a pin (hinge) in the center of the left span, making the beam statically determinate. There is continuous lateral support. The concentrated loads are service live loads. Determine whether a W18 × 60 of A992 steel is adequate.

a. Use LRFD.

b. Use ASD.

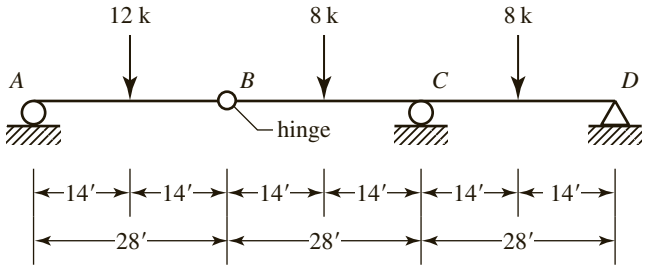


FIGURE P5.5-5

5.5-6 A W12 × 30 of A992 steel has an unbraced length of 10 feet. Using $C_b = 1.0$,

a. Compute L_p and L_r. Use the equations in Chapter F of the AISC Specification. Do not use any of the design aids in the *Manual*.

b. Compute the flexural design strength, $\phi_b M_n$.

c. Compute the allowable flexural strength M_n/Ω_b.

5.5-7 A W18 × 46 is used for a beam with an unbraced length of 10 feet. Using $F_y = 50$ ksi and $C_b = 1$, compute the nominal flexural strength. Use the AISC equations in Chapter F of the Specification. Do not use any of the design aids in the *Manual*.

5.5-8 A W18 × 71 is used as a beam with an unbraced length of 9 feet. Use $F_y = 65$ ksi and $C_b = 1$ and compute the nominal flexural strength. Compute everything with the equations in Chapter F of the AISC Specification.

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5.5-9 The beam shown in Figure P5.5-9 is a W36 × 182. It is laterally supported at *A* and *B*. The 300 kip load is a service live load. Using the unfactored service loads,

a. Compute C_b. Do not include the beam weight in the loading.

b. Compute C_b. Include the beam weight in the loading.

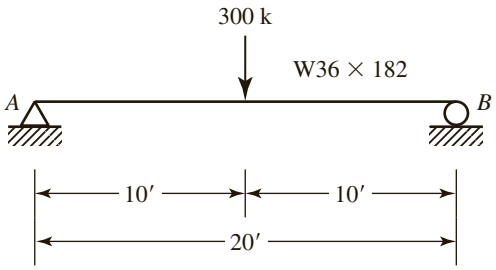


FIGURE P5.5-9

5.5-10 If the beam in Problem 5.5-9 is braced at *A*, *B*, and *C*, compute C_b for the unbraced length *AC* (same as C_b for unbraced length *CB*). Do not include the beam weight in the loading.

a. Use the unfactored service loads.

b. Use factored loads.

5.5-11 The beam shown in Figure P5.5-11 has lateral support at *a*, *b*, *c*, and *d*. Compute C_b for segment *b*–*c*.

a. Use the unfactored service loads.

b. Use factored loads.

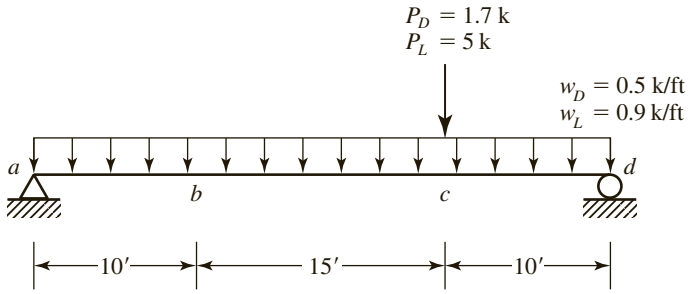


FIGURE P5.5-11

5.5-12 A W21 × 68 of A992 steel is used as a simply supported beam with a span length of 50 feet. The only load in addition to the beam weight is a uniform live load. If lateral support is provided at 10-foot intervals, what is the maximum service live load, in kips/ft, that can be supported?

a. Use LRFD.

b. Use ASD.

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5.5-13 The beam shown in Figure P5.5-13 is laterally braced only at the ends. The 30-kip load is a service live load. Use $F_y = 50$ ksi and determine whether a W14 × 38 is adequate.

a. Use LRFD.

b. Use ASD.

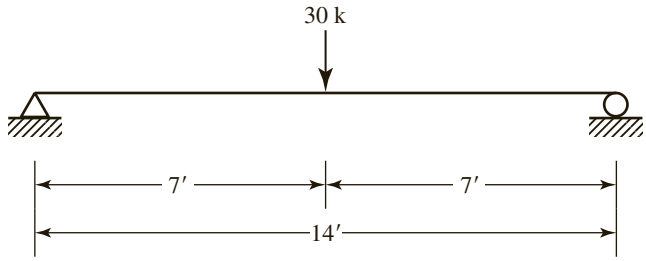


FIGURE P5.5-13

5.5-14 Repeat Problem 5.5-13 for an MC18 × 58 (Assume that the load is applied through the shear center so that there is no torsional loading.) Use $F_y = 36$ ksi

5.5-15 Determine whether a W24 × 104 of A992 steel is adequate for the beam shown in Figure P5.5-15. The uniform load does not include the weight of the beam. Lateral support is provided at *A*, *B*, and *C*.

a. Use LRFD.

b. Use ASD.

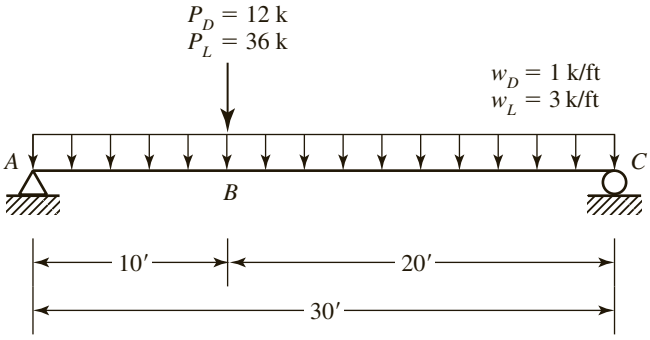


FIGURE P5.5-15

5.5-16 The beam shown in Figure P5.5-16 is laterally braced at *A*, *B*, *C*, and *D*. Is a W14 × 132 adequate for $F_y = 50$ ksi?

a. Use LRFD.

b. Use ASD.

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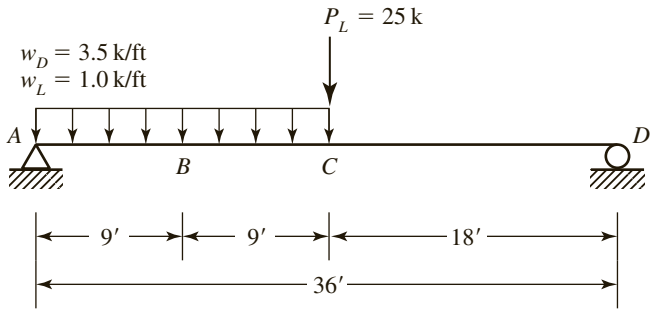


FIGURE P5.5-16

Bending Strength of Noncompact Shapes

5.6-1 A W12 × 65 is used as a simply supported, uniformly loaded beam with a span length of 50 feet and continuous lateral support. The yield stress, F_y, is 50 ksi. If the ratio of live load to dead load is 3, compute the available strength and determine the maximum total service load, in kips/ft, that can be supported.

a. Use LRFD.

b. Use ASD.

5.6-2 A W14 × 99 of A992 steel is used as a beam with lateral support at 10-foot intervals. Assume that $C_b = 1.0$ and compute the nominal flexural strength.

5.6-3 A built-up shape consisting of two ¾ × 18 flanges and a ¾ × 52 web is used as a beam. If A572 Grade 50 steel is used, what is the nominal flexural strength based on flange local buckling? For width–thickness ratio limits for welded shapes, refer to Table B4.1b in Chapter B of the AISC Specification, "Design Requirements."

5.6-4 A built-up shape consisting of two ¾ × 16 flanges and a ½ × 40 web is used as a beam with continuous lateral support. If A572 Grade 50 steel is used, what is the nominal flexural strength? For width–thickness ratio limits for welded shapes, refer to Table B4.1b in Chapter B of the AISC Specification, "Design Requirements."

Shear Strength

5.8-1 Compute the nominal shear strength of an M10 × 7.5 of A572 Grade 65 steel.

5.8-2 Compute the nominal shear strength of an M12 × 11.8 of A572 Grade 65 steel.

5.8-3 The beam shown in Figure P5.8-3 is a W16 × 31 of A992 steel and has continuous lateral support. The two concentrated loads are service live loads. Neglect the weight of the beam and determine whether the beam is adequate.

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a. Use LRFD.

b. Use ASD.

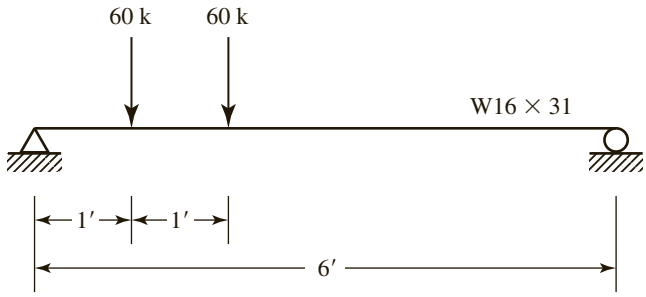


FIGURE P5.8-3

5.8-4 The cantilever beam shown in Figure P5.8-4 is a W16 × 45 of A992 steel. There is no lateral support other than at the fixed end. Use an unbraced length equal to the span length and determine whether the beam is adequate. The uniform load is a service dead load that includes the beam weight, and the concentrated load is a service live load.

a. Use LRFD.

b. Use ASD.

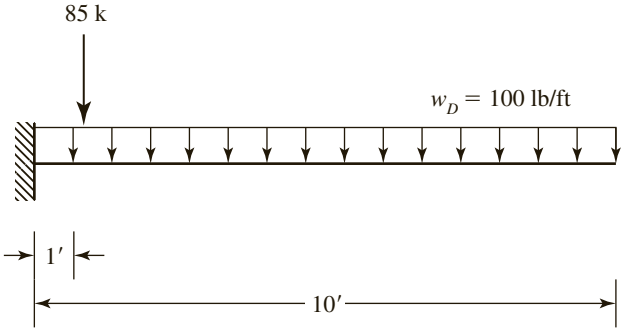


FIGURE P5.8-4

Design

5.10-1 Use A992 steel and select a W-shape for the following beam:

- Simply supported with a span length of 30 feet
- Laterally braced only at the ends
- Service dead load = 0.75 kips/ft
- The service live load consists of a 34-kip concentrated load at the center of the span

There is no limit on the deflection.

a. Use LRFD.

b. Use ASD.

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5.10-2 Use A992 steel and select the most economical W shape for the beam in Figure P5.10-2. The beam weight is not included in the service loads shown. Do not check deflection. Assume continuous lateral support.

a. Use LRFD.

b. Use ASD.

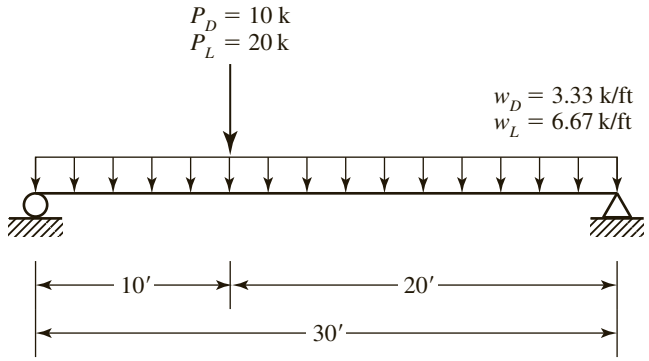


FIGURE P5.10-2

5.10-3 Same as Problem 5.10-2, except that lateral support is provided only at the ends and at the concentrated load.

5.10-4 The beam shown in Figure P5.10-4 has lateral support only at the ends. The uniform load is a *superimposed* dead load, and the concentrated load is a live load. Use A992 steel and select a W-shape. The *live load* deflection must not exceed $L/360$.

a. Use LRFD.

b. Use ASD.

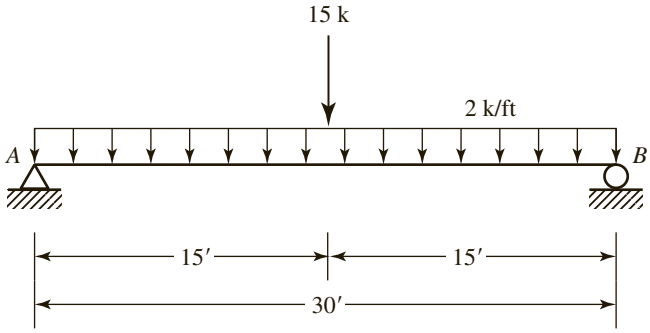


FIGURE P5.10-4

5.10-5 The given beam is laterally supported at the ends and at the $\frac{1}{3}$ points (points 1, 2, 3, and 4). The concentrated load is a service live load. Use F_y = 50 ksi and select a W-shape. Do not check deflections.

a. Use LRFD.

b. Use ASD.

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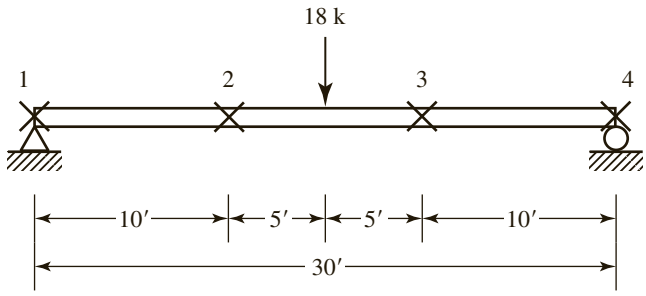


FIGURE P5.10-5

5.10-6 The beam shown in Figure P5.10-6 has lateral support at the ends only. The concentrated loads are live loads. Use A992 steel and select a shape. Do not check deflections. Use $C_b = 1.0$ (this is conservative).

a. Use LRFD.

b. Use ASD.

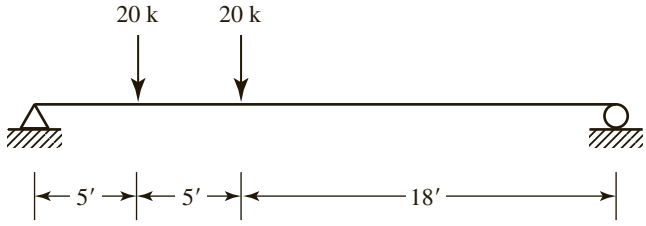


FIGURE P5.10-6

5.10-7 The beam shown in Figure P5.10-7 is part of a roof system. Assume that there is partial lateral support equivalent to bracing at the ends and at mid-span. The loading consists of 170 lb/ft dead load (not including the weight of the beam), 100 lb/ft roof live load, 280 lb/ft snow load, and 180 lb/ft wind load acting *upward*. The dead, live, and snow loads are gravity loads and always act downward, whereas the wind load on the roof will always act upward. Use A992 steel and select a shape. The total deflection must not exceed $L/180$.

a. Use LRFD.

b. Use ASD.

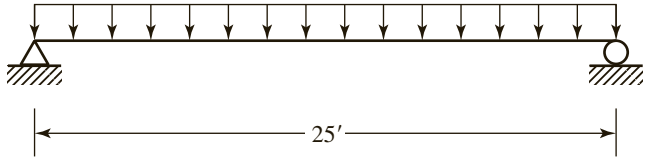


FIGURE P5.10-7

Manual Table 6-2 for Beam Analysis and Design

5.11-1 Use *Manual* Table 6-2 and ASD to determine whether a W18 × 55 has enough moment strength for the following conditions:

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Simply supported span length = 20 ft

C_b = 1.14

Required unfactored moment strength, M_a, is 275 ft-kips

F_y = 50 ksi

Important: Clearly outline your procedure.

5.11-2 Use *Manual* Table 6-2 and LRFD to select a W shape for the following conditions:

Simply supported span length = 30 ft

Lateral support only at the ends

The loading consists of a concentrated live load of 25 kips at mid-span and a uniform dead load (not including the beam weight) of 3 kips/ft

The *live load* deflection must not exceed $L/360$

A992 steel is used

Important: Clearly outline your procedure.

Floor and Roof Framing Systems

5.11-3 Use F_y = 50 ksi and select a shape for a typical floor beam AB. Assume that the floor slab provides continuous lateral support. The maximum permissible live load deflection is $L/180$. The service dead loads consist of a 5-inch-thick reinforced-concrete floor slab (normal weight concrete), a partition load of 20 psf, and 10 psf to account for a suspended ceiling and mechanical equipment. The service live load is 60 psf.

a. Use LRFD.

b. Use ASD.

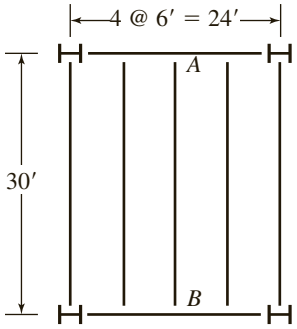


FIGURE P5.11-3

5.11-4 Select a W-shape for the following conditions:

Beam spacing = 5 ft-6 in.

Span length = 30 ft

Slab thickness = 4½ in. (normal-weight concrete)

Partition load = 20 psf

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Weight of ceiling = 5 psf
Live load = 150 psf
F_y = 50 ksi

The maximum live load deflection cannot exceed $L/360$.

a. Use LRFD.

b. Use ASD.

5.11-5 Select a W-shape for the following conditions:

Beam spacing = 12 ft
Span length = 25 ft
Slab and deck combination weight = 43 psf
Partition load = 20 psf
Ceiling weight = 5 psf
Flooring weight = 2 psf
Live load = 160 psf
F_y = 50 ksi

The maximum live load deflection cannot exceed $L/360$.

a. Use LRFD.

b. Use ASD.

5.11-6 Select a W-shape for the following conditions:

Beam spacing = 10 ft
Span length = 20 ft
Slab and deck weight = 51 psf
Partition load = 20 psf
Miscellaneous dead load = 10 psf
Live load = 80 psf
F_y = 50 ksi

The maximum live load deflection cannot exceed $L/360$.

a. Use LRFD.

b. Use ASD.

5.11-7 Select an A992 W-shape for beam AB of the floor system shown in Figure P5.11-7. In addition to the weight of the beam, the dead load consists of a 5-inch-thick reinforced concrete slab (normal-weight concrete). The live load is 80 psf, and there is a 20-psf partition load. The total deflection must not exceed $L/240$.

a. Use LRFD.

b. Use ASD.

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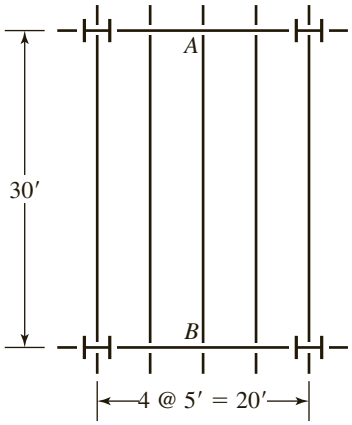


FIGURE P5.11-7

5.11-8 Use LRFD and design a typical girder for the floor system of Problem 5.11-7. Do not check deflections. Assume that the girder is supporting beams on each side, and assume that the beams weigh 35 lb/ft. Consider the beam reactions to act as point loads on the girder.

a. Use LRFD.

b. Use ASD.

5.11-9 Same as Problem 5.11-8, but let all the loads on the girder act as a uniform load (be sure to include the weight of the beams).

Holes in Beams

5.12-1 A W16 × 31 of A992 steel has two holes in each flange for $\frac{7}{8}$-inch-diameter bolts.

a. Assuming continuous lateral support, verify that the holes must be accounted for and determine the nominal flexural strength.

b. What is the percent reduction in strength?

5.12-2 A W21 × 48 of A992 steel has two holes in each flange for $\frac{3}{4}$-inch-diameter bolts.

a. Assuming continuous lateral support, verify that the holes must be accounted for and determine the nominal flexural strength.

b. What is the percent reduction in strength?

5.12-3 A W18 × 35 of A992 steel has two holes in the tension flange for $\frac{3}{4}$-inch-diameter bolts.

a. Assuming continuous lateral support, verify that the holes must be accounted for and determine the reduced nominal flexural strength.

b. What is the percent reduction in strength?

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Open-Web Steel Joists

5.13-1 A floor system consists of open-web steel joists spaced at 3 feet and spanning 20 feet. The live load is 80 psf, and there is a 4-inch-thick normal-weight reinforced concrete floor slab. Other dead load is 5 psf. Assume that the slab provides continuous lateral support. Use Figure 5.35 and select a K-series joist.

5.13-2 Use Figure 5.35 and select an open-web steel joist for the following floor system. The span length is 22 feet and the joist spacing is 4 feet. The loads consist of a 40 psf live load, a partition load of 20 psf, a slab and metal deck system weighing 32 psf, and a ceiling and light fixture weight of 5 psf. Assume that the slab provides continuous lateral support. Is there likely to be a deflection problem? Why or why not?

Beam Bearing Plates and Column Base Plates

5.14-1 A W14 × 61 must support a concentrated service live load of 85 kips applied to the top flange. Assume that the load is at a distance of at least half the beam depth from the support and design a bearing plate. Use $F_y = 50$ ksi for the beam and $F_y = 36$ ksi for the plate.

a. Use LRFD.

b. Use ASD.

5.14-2 Design a bearing plate of A36 steel to support a beam reaction consisting of 28 kips dead load and 56 kips live load. Assume that the bearing plate will rest on concrete with a surface area larger than the bearing area by an amount equal to 1 inch of concrete on all sides of the plate. The beam is a W30 × 99 with $F_y = 50$ ksi, and the concrete strength is $f'_c = 3$ ksi.

a. Use LRFD.

b. Use ASD.

5.14-3 Design a base plate for a W12 × 87 column supporting a service dead load of 65 kips and a service live load of 195 kips. The support will be a 16-inch × 16-inch concrete pier. Use A36 steel and $f'_c = 3.5$ ksi.

a. Use LRFD.

b. Use ASD.

5.14-4 Design a column base plate for a W10 × 33 column supporting a service dead load of 20 kips and a service live load of 50 kips. The column is supported by a 12-inch × 12-inch concrete pier. Use A36 steel and $f'_c = 3$ ksi.

a. Use LRFD.

b. Use ASD.

Biaxial Bending

5.15-1 A W18 × 55 is loaded as shown in Figure P5.15-1, with forces at midspan that cause bending about both the strong and weak axes. The loads shown are service loads, consisting of equal parts dead load and live load. Determine whether the AISC Specification is satisfied. The steel is A572 Grade 50, and lateral bracing is provided only at the ends. *Manual* Table 6-2 may be used.

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a. Use LRFD.

b. Use ASD.

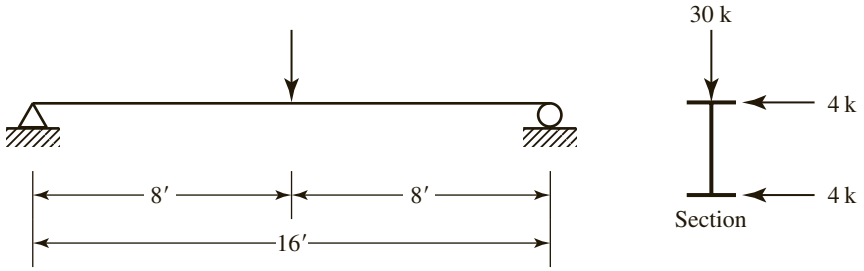


FIGURE P5.15-1

5.15-2 The 24-kip concentrated load shown in Figure P5.15-2 is a service live load. Neglect the weight of the beam and determine whether the beam satisfies the AISC Specification if A992 steel is used. Lateral support is provided at the ends only. *Manual* Table 6-2 may be used.

a. Use LRFD.

b. Use ASD.

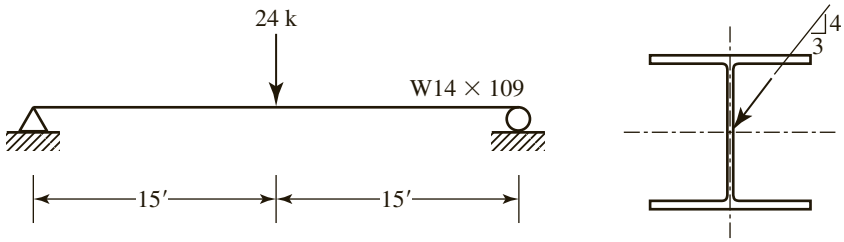


FIGURE P5.15-2

5.15-3 The beam shown in Figure P5.15-3 is a W21 × 68 of A992 steel and has lateral support only at the ends. Check it for compliance with the AISC Specification. *Manual* Table 6-2 may be used.

a. Use LRFD.

b. Use ASD.

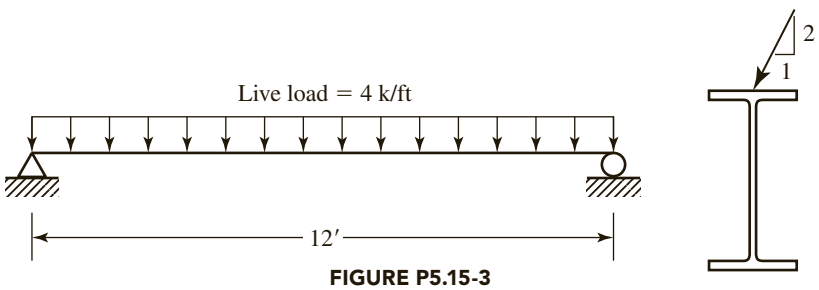


FIGURE P5.15-3

5.15-4 Check the beam shown in Figure P5.15-4 for compliance with the AISC Specification. Lateral support is provided only at the ends, and A992 steel

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is used. The 15-kip service loads are 30% dead load and 70% live load. *Manual* Table 6-2 may be used.

a. Use LRFD.

b. Use ASD.

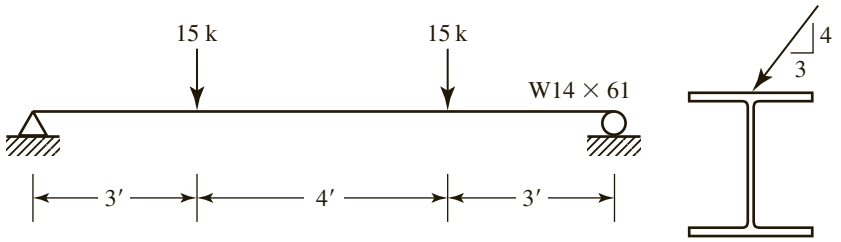


FIGURE P5.15-4

5.15-5 The beam shown in Figure P5.15-5 is simply supported and has lateral support only at its ends. Neglect the beam weight and determine whether it is satisfactory for each of the loading conditions shown. A992 steel is used, and the 1.2 kip/ft is a service live load. *Manual* Table 6-2 may be used.

a. Use LRFD.

b. Use ASD.

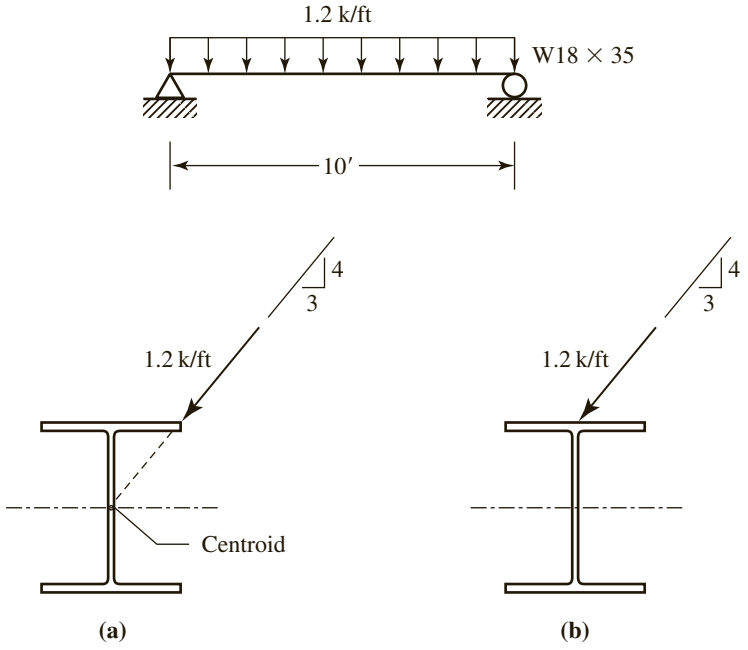


FIGURE P5.15-5

5.15-6 The truss shown in Figure P5.15-6 is part of a roof system supporting a total gravity load of 40 psf of roof surface, half dead load and half snow.

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Trusses are spaced at 10 feet on centers. Assume that wind load is not a factor and investigate the adequacy of a W6 × 12 of A992 steel for use as a purlin. No sag rods are used, so lateral support is at the ends only. *Manual* Table 6-2 may be used.

a. Use LRFD.

b. Use ASD.

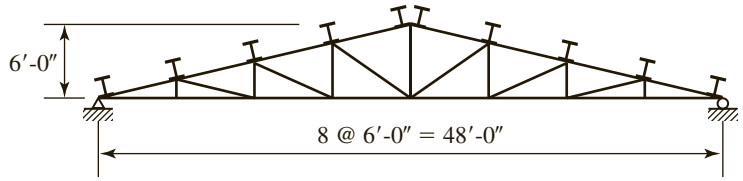


FIGURE P5.15-6

5.15-7 The truss shown in Figure P5.15-7 is one of several roof trusses spaced 18 feet apart. Purlins are located at the joints and halfway between the joints. Sag rods are located midway between the trusses. The weight of the roofing materials is 15 psf, and the snow load is 20 psf of horizontal projection of the roof surface. Use LRFD and select a W shape of A992 steel for the purlins. *Manual* Table 6-2 may be used.

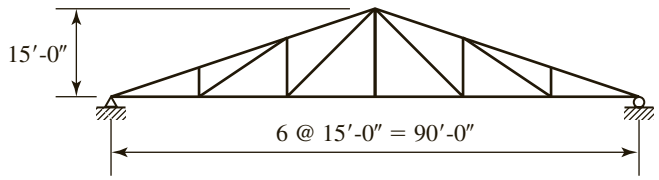


FIGURE P5.15-7

5.15-8 Same as Problem 5.15-7, except that the sag rods are at the third points.

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Many beams and columns, such as the ones in this structural steel frame, are subjected to significant amounts of both bending and compression. These members are called beam-columns.

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