Chapter Two

Layout: **10 Hrs.**

1. Introduction.
2. Pulse Code Modulation (PCM).
3. Differential Pulse Code Modulation (DPCM).
4. Delta modulation.
5. Adaptive delta modulation.
6. Sigma Delta Modulation (SDM).
7. Linear Predictive Coder (LPC).
8. **MATLAB programs.**

Lecture Four

Quantization

Objective of Lecture:

Understand quantization technique which is sliced the amplitude signal into pieces.

Behavioral goals:

Student will be able to convert analog amplitude signal to discrete amplitude signal and dedicated unique value for each sampled which finally convert the signal to the unique numbers.

This lecture answer important questions which are:

What is quantization technique mean?

Why quantization technique is important?

How is quantization technique done?

What is quantization technique do for the signal?

What problems in quantization process?

2.3. Quantization Process

Sampling converts a continuous time signal into discrete time signal by measuring the signal value at regular interval of the time called T_s , in otherworld, multiplying the signal by train of samples. This known as discretization of signal along the time axis or horizontal axis.

Quantization is converts a continuous amplitude signal into a discreet amplitude signal by dividing the amplitude axis (y-axis) into several uniformly spaced/non-uniformly spaced amplitude level called quantization level. This is known as discretization along amplitude axis or vertical axis. ***Rounding and truncation*** *are typical examples of quantization processes.*

Quantization reshapes (reform) each sample height in voltage or in current mode only. Here we get defined levels of voltage/current with respect to defined instants of time (define sample amplitude with respect to their position in time T_s). That's why this signal obtained after quantization is called as **Discrete Time Discrete Amplitude Signal**. In other word, the simplest definition of quantization is quantize (partitions) a signal is to choose the digital amplitude value closest to the original analog amplitude.

We confine attention to a quantization process in this chapter that is *memoryless* and *instantaneous*, which means that the transformation at time is not affected by earlier or later samples of the message signal. This form of quantization, though not optimal, is commonly used in practice because of its simplicity.

Quantizers can be of a *uniform* or *non-uniform* type. In a uniform quantizer, the representation levels are uniformly spaced (levels or step size are equally sliced); otherwise, the quantizer is non-uniform. The quantizers considered in this section are of the **uniform** as shown in figure 2.9.

2.3.1 SNR of Quantization Process

SNR is important metric of communication system performance measurement. If the signal $g(t)$ has dynamic (variant or analog) range amplitude from A_m to $-A_m$ or simply from 0 to A_m (where, A_m message amplitude or information signal amplitude), then the uniform qauntizer can be divided into M zone or regions of the same size call step-size (or slices) S which is given as:

$$S_k = \frac{2\,A_m}{M} \text{ or } \frac{A_m}{M} \tag{1}$$

Where, $S_k : \{S_k < A_m < S_{k+1}\}$, $k = 1,2,3, \dots\dots, M$. The amplitudes, S_k are called *decision levels* or *decision thresholds*. At the qauntizer output, the index k is transformed into an amplitude that represents all amplitudes that lie inside the interval S_k.

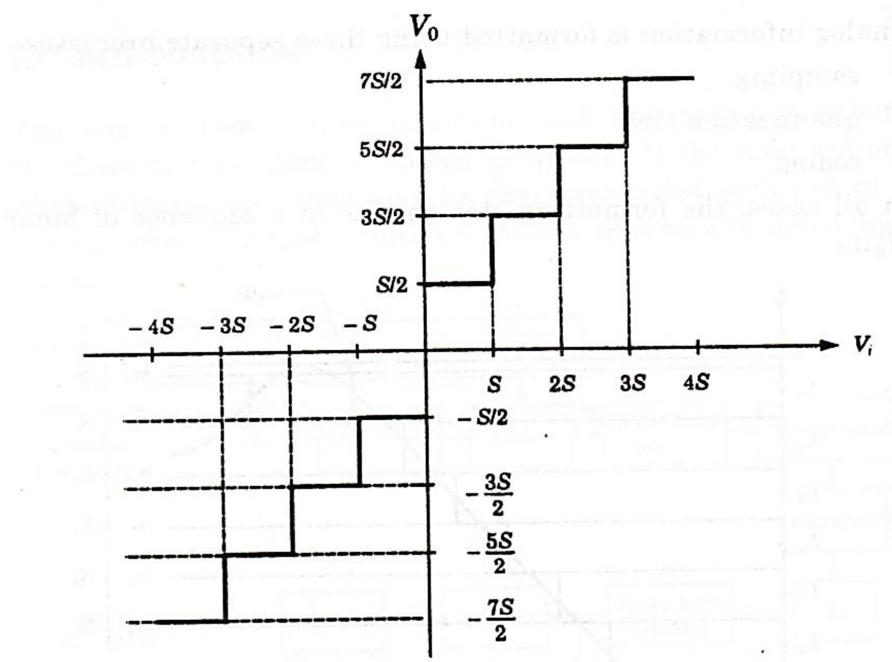

Figure 2.9. Quantization steps

The difference between input and output single of qauntizer is called qauntizer error or quantization noise. If the input signal is assumed to be random signal, the qauntizer error q_e varies randomly within the interval $-\frac{S}{2} < q_e < \frac{S}{2}$ or $0 < q_e < S$. The probability

distribution function PDF (assuming the probability of error is equally likely in anywhere inside the range $0 < q_e < S$) of this quantization error can be written as:

$$P_{q_e}(q_e) = 1 \qquad for\ -\frac{S}{2} < q_e < \frac{S}{2}$$
$$P_{q_e}(q_e) = 0 \qquad elsewhere$$

(2)

The quantization noise power (Mean Squared Value = $E(q_e^2) = N_q$, error is part of S, hence average is dividing by total S) of quantization error N_q is given as:

$$N_q = \frac{1}{S}\int_{-S/2}^{+S/2} {q_e}^2\ P_{q_e}(q_e)\ d_{q_e} \ \ or\ N_q = \frac{1}{S}\int_{0}^{+S} {q_e}^2\ P_{q_e}(q_e)\ d_{q_e}$$

$$N_q = \frac{1}{S}\left[\frac{q_e}{3}\right]_{-\frac{S}{2}}^{+\frac{S}{2}} = \frac{S^2}{12}$$

(3)

Suppose any signal amplitude A = $-A_m\ to + A_m = 2A_m\ (peak\ to\ peak)$. Assuming that the number of representation levels (quantization levels) M, then the relationship between A and M quantization level is given as:

$$2A_m = MS \quad or\ S = \frac{2A_m}{M}$$

(4)

Substitute (4) in (3), the average quantization noise power is given as:

$$N_q = \frac{S^2}{12} = \frac{4\ A_m^2}{12\ M^2} = \frac{A_m^2}{3\ M^2}$$

(5)

Average signal power is given as:

$$S_{av}\ or\ P_{av} = \frac{A_m^2}{2}$$

(6)

Hence, signal to noise ratio ($\frac{S_q}{N_q}$) is given as:

$$(SNR)_q = \frac{S_q}{N_q} = \frac{A_m^2}{2} \times \frac{3\, M^2}{A_m^2} = \frac{3}{2}\, M^2$$

(7)

To represent $M\ level,$ $N\ bits$ are required (relationship between $M\ level\ and\ N\ bits$ described later on), hence:

$$M = 2^N$$

$$\frac{S_q}{N_q} = \frac{3}{2}\, 2^{2N}$$

(8)

Expressing the (8) in dB, we get

$$(SNR)_q = 10 \log \frac{3}{2}\, 2^{2N}$$

$$(SNR)_q = 10 \log \frac{3}{2} + 20 \log 2^N$$

$$(SNR)_q = 1.8 + 6.02\, N$$

(9)

Signal to noise ratio ($\frac{S_q}{N_q}$) as function of $M\ levels$ is given as:

$$(SNR)_q = 10 \log_{10} \left(\frac{3}{2} M\right)$$

(10)

It clear from equation 10, if the number of level (step-size or slices) increases, the signal to noise ratio increase because as number of slices increases the distances between M levels reduces and quantization error reduces as well which cumulatively improve signal to noise ratio. In fact, increasing quantization level has drawback which is increases number of bits per sample then increases number of bits that represent quantized signal and increasing number of bits required higher channel bandwidth which is scary factor in communication.

Also increasing number of M increase the complexity of quantizer. Another problem is synchronization between levels.

Also, if the step size increases the quantization error increases as well due to quantization error directly proportional to step size. Where, if the step size increased, the sample may fall in the mid of the step size which result high error after making round up or down.

The error associated with quantization is probabilistic. Since the quantization is merely round-off, if the signal has equal of being any value within particular quantization region, then the error is equally likely to be any value ranging from zero to plus minus half the quantization region. We can therefor represent error as uniformly distributed random variable. The maximum error in using uniform quantization is:

$$q_e = \pm 0.5 \frac{dynamic\ range\ of\ the\ signal}{2^N} = \frac{dynamic\ range\ of\ signal}{2^{N+1}}$$

$$q_e = \pm 0.5 \frac{2A_m}{M} = \pm 0.5\ S \tag{11}$$

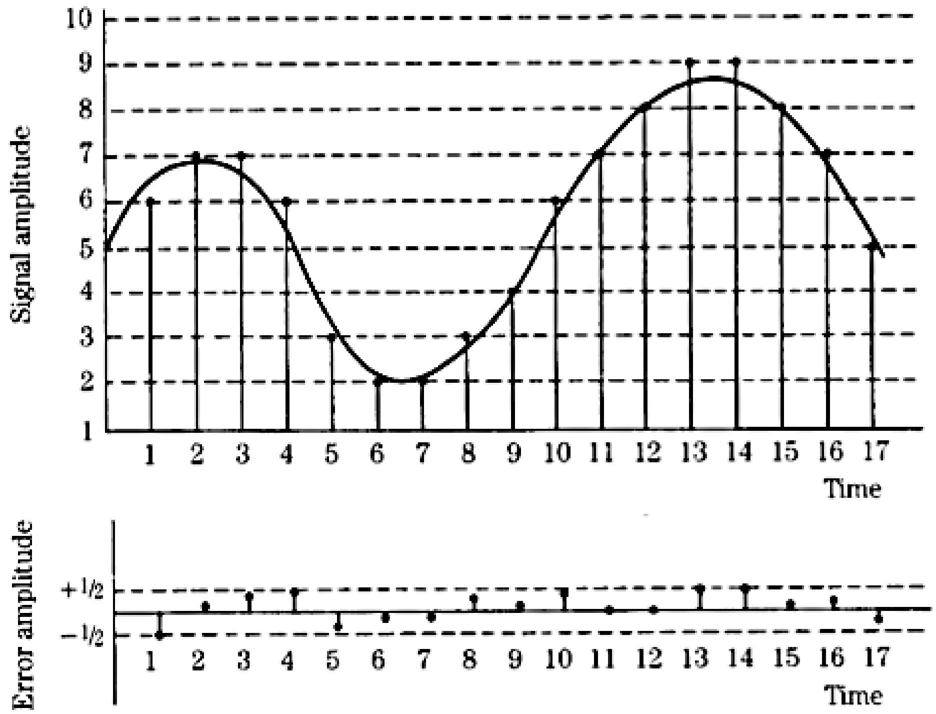

Figure 2.10: Quantization and quantization error, where $M = 8$, then $N = \log_2 8 = 3\ bits$.