Chapter Two

Layout: **10 Hrs.**

1. Introduction.
2. Pulse Code Modulation (PCM).
3. Differential Pulse Code Modulation (DPCM).
4. Delta modulation.
5. Adaptive delta modulation.
6. Sigma Delta Modulation (SDM).
7. Linear Predictive Coder (LPC).
8. **MATLAB programs.**

Lecture Five

Pulse Code Modulation (Pulse to Code Conversion)

Objective of Lecture:

Understand the way by which we convert the analog signal to binary bits.

Behavioral goals:

Student will be able to convert discrete time and amplitude signal to binary.

This lecture answer important questions which are:

What is PCM?

Why PCM is important?

How is PCM done?

Where can you exploit PCM?

What are the problems in PCM?

2.4. Pulse Code modulation (PCM)

PCM (waveform coder) is process by which the analog signal is transferred to digital binary form (bits) to be transmitted over baseband channel (physical channel or guided channel such as coaxial cable), see Fig. 2.11. With the sampling and quantization processes at our disposal (في متناول اليد), we are now ready to describe pulse-code modulation, which is the most basic form of digital pulse modulation.

In *pulse code modulation (PCM), a message signal is represented by a sequence of coded binary, which is accomplished by representing the signal in binary form after converting analog signal to discrete form in both time and amplitude.*

Actually, there are three types of information, digital (bits), textual and analog information. In this chapter analog information are considered. PCM block diagram in baseband signal transmission shown in Fig. 2.11, which is name as formatting, and it consist sampling, quantization and encoding.

2.4.1. Pulse Code modulation (PCM) Transmitter

The basic operations performed in the transmitter of a PCM system are *sampling*, *quantization*, and *encoding*, as shown in Fig. 2.11; the low-pass filter prior to sampling is included merely to prevent aliasing of the message signal. The quantizing and encoding operations are usually performed in the same circuit, which is called an *analog-to-digital converter ADC.*

Sampler and quantizer are already explained in the previous section. **Encoding** is technique by which each step-size in quantization translated to binary code which is named as **codeword**. <u>Encoding is done as follow</u>: suppose that, in a binary code, each codeword consists of ***N bits***: the bit is an acronym (رمز) for *binary digit*. Then *N* denotes the number of *bits per sample M (*per step-size or quantization level M*)*. Hence, by using such a code, we represent a total of 2^N distinct numbers. For example, a signal quantized into one of

256 levels which is denoted as M may be represented by an 8-bit code word (i.e. $N = \log_2 M$, $N = \log_2 256$, $N = \log_2 2^8 = 8\, bits$).

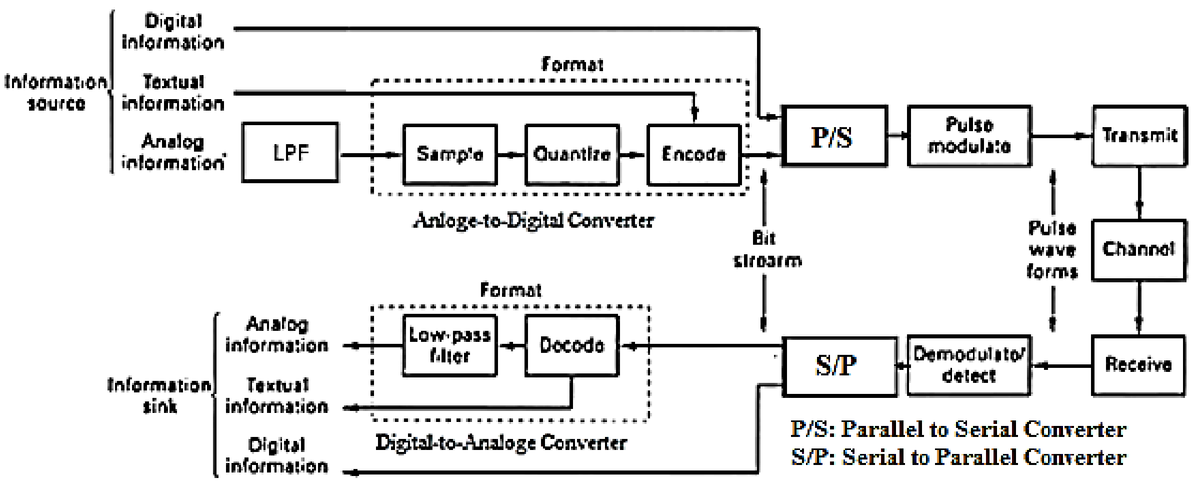

Figure 2.11. Basic elements of digital baseband communication system (PCM transmitter included in diagram as format block)

2.4.2. Pulse Code modulation (PCM) Receiver

The serial bits converted to parallel bits using serial to parallel converter, then regrouped bits and converted to analog signal using digital to analog converter. Then the final operation, to recover the original analog signal is done by passing signal through low pass LP reconstruction filter, the filter interpolates (estimation of samples from received converted sampled signal) the signal between samples.

2.4.3. Transmission Bandwidth (Bit Rate) for PCM

Bandwidth is important metric of communication system performance measurement. A signal $m(t)$ bandlimited to W Hz, in the sequel, sampling rate f_s required

is $2W\ sample/sec$, if each quantized samples encoded to $N\ bits$, then total channel bandwidth required is given as

$$B_{PCM} = N \times 2W\ bits/sec \tag{1}$$

Then, we rewrite 13 as:

$$B_{PCM} = N\ f_s = f_s \times \log_2 M\ \ bits/\sec\ or\ Hz \tag{2}$$

From 13, minimum bandwidth required for PCM is proportional to the message signal bandwidth and number of bit per quantization level or it is depend on quantization level.

Exercise 2.3: Use PCM to convert the given signal (see 2.13) into binary form? Then find SNR of quantization technique using 16 $quantization\ levels$? If the signal bandlimited to 250 Hz sampling rate is 8000 $sample\ per\ second$, find PCM bandwidth transmission? Draw block diagram of the system?

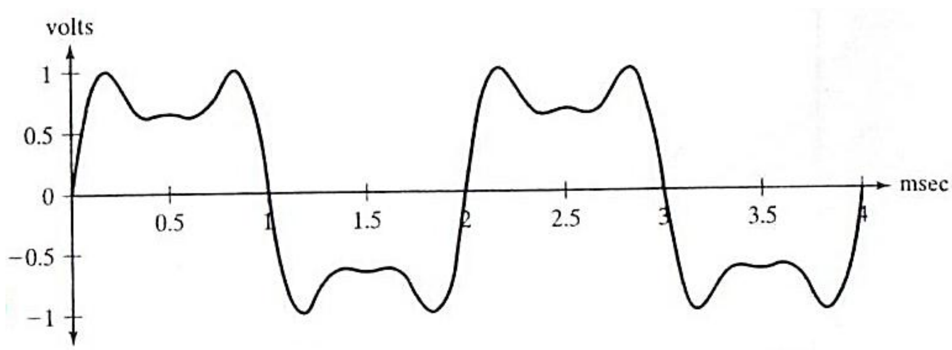

Figure 2.13. Exercise 2.3

Solution:

Number of quantization levels M is 16, hence we divide the original signal into 16 slice, then number of bits per each slice or per each quantization level is:

$$N = \log_2 M = \log_2 16 = \log_2 2^4 = 4\ bits/sample$$

The signal to noise ratio of quantization process is

$$SNR = \frac{3}{2}\,2^{2N} = \frac{3}{2}\,2^{2\times 4} = 384$$

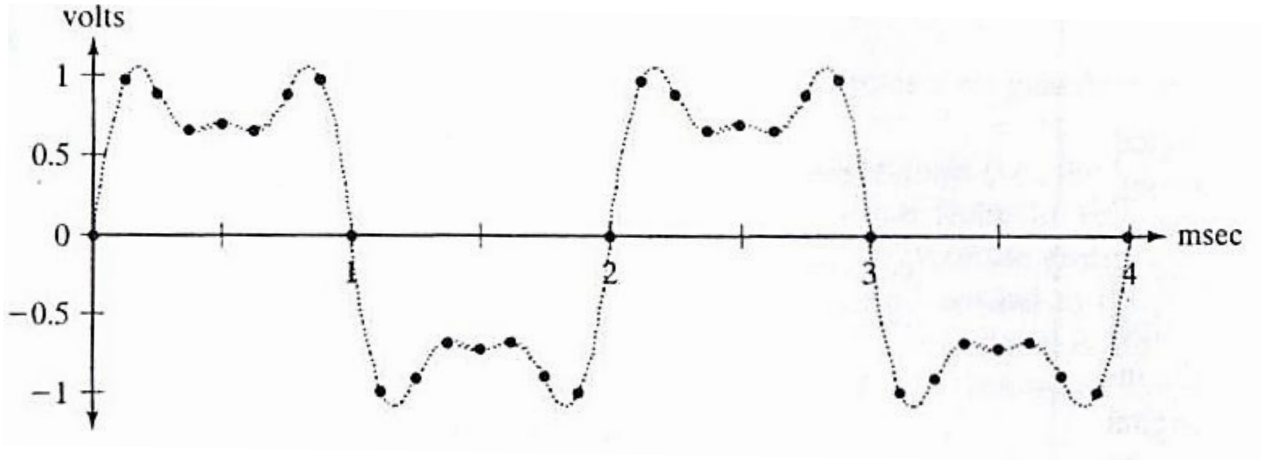

Figure 2.12. Exercise 2.3, after sampling.

The bandwidth transmission required is

$$B_{PCM} = N \times f_s = 8000 \times 4 = 32000\ bps$$

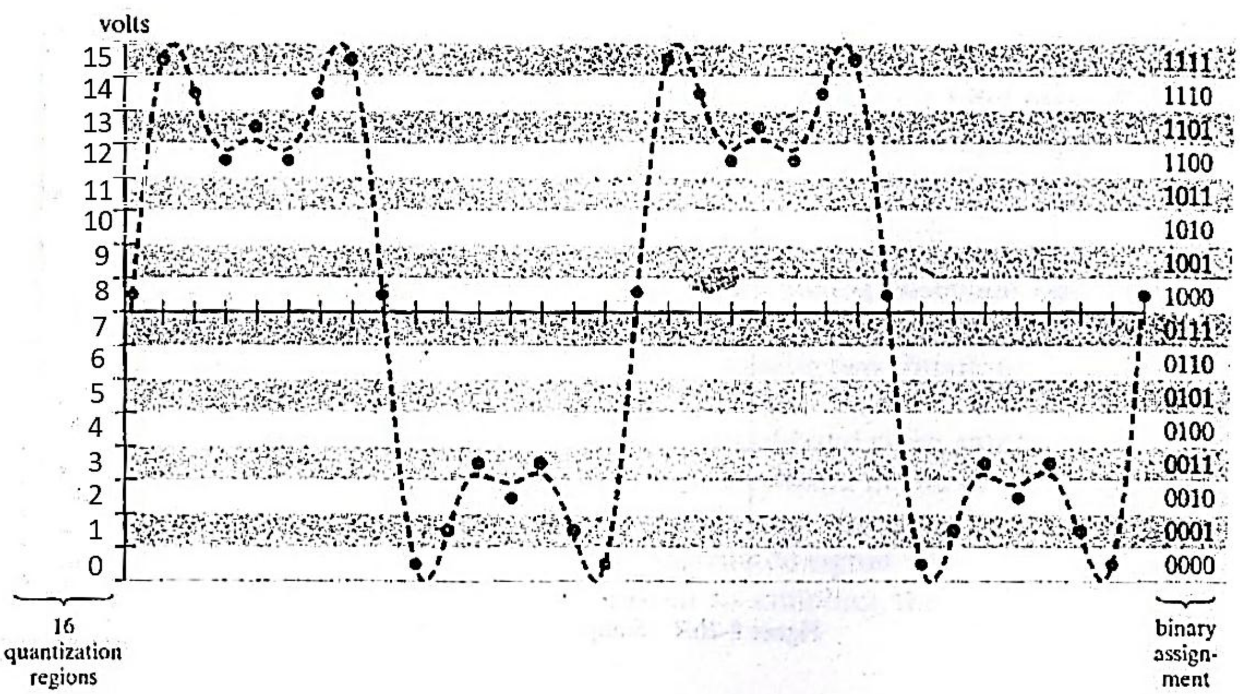

Figure 2.13. Exercise 2.3, sampling, quantization and encoding.

Binary Number System for N = 4

Ordinal Number of Representation Level M	Level Number Expressed as Sum of Powers of 2	Codeword
0		0000
1	2^0	0001
2	2^1	0010
3	$2^1 + 2^0$	0011
4	2^2	0100
5	$2^2 + 2^0$	0101
6	$2^2 + 2^1$	0110
7	$2^2 + 2^1 + 2^0$	0111
8	2^3	1000
9	$2^3 + 2^0$	1001
10	$2^3 + 2^1$	1010
11	$2^3 + 2^1 + 2^0$	1011
12	$2^3 + 2^2$	1100
13	$2^3 + 2^2 + 2^0$	1101
14	$2^3 + 2^2 + 2^1$	1110
15	$2^3 + 2^2 + 2^1 + 2^0$	1111

Encoding the obtain for example level 3 and 7

$$Level\ 3 = 0 \times 2^3 + \ 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3$$

$$Level\ 8 = 1 \times 2^3 + \ 1 \times 2^2 + \ 1 \times 2^1 \ + 1 \times 2^0 = 8$$

Reminder levels have been listed in the table above.

Exercise 2.4: A telephone signal bandlimited to 4 kHz is to be transmitted by PCM, the signal to quantization noise is to be at least 40 dB. Find the level into which the signal is to be encoded? Find the bandwidth of PCM?

Solution: Given

$$SNR\ in\ dB = \ \ 1.8 + 6\,N = 40\ dB$$

$$N = \ \frac{40 - 1.8}{6} = 6.34 \ \approx 7\frac{bits}{samples}$$

Number of the levels then $M = 2^7 = 128\ levels\ or\ steps\ or\ slices$

Transmission bandwidth required:

$$B_{PCM} = 2\,f_m\,N = 2 \times 4 \times 10^3 \times 7 = 56\ kHz$$

Exercise 2.5: given audio signal with spectral components in the range of 300-3400 Hz, the sampling rate being 8 kHz, draw the block diagram of the PCM system indicating the specification of each block, the required SNR to be at least 30 dB.

Solution:

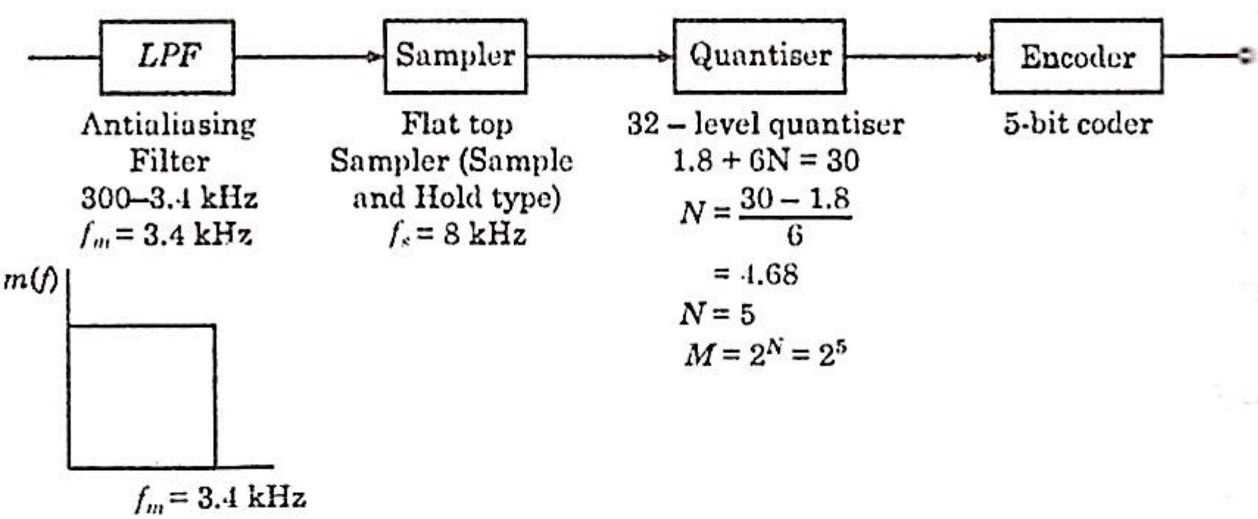

Figure 2.14 solution of exercise 2.6

Exercise 2.6: signal m_1 is bandlimited to 5 kHz and 3 other signals m_2, m_3 and m_4 each bandlimited to 4 kHz are time multiplexed and fed to a PCM encoder having $M = 1024$ quantization levels. Find bit rate of PCM and also minimum theoretical bandwidth for transmission?

Solution:

$signals$	f_m	f_s
m_1	$5\ kHz$	$10\ kHz$

m_2	$4\ kHz$	$8\ kHz$
m_3	$4\ kHz$	$8\ kHz$
m_4	$4\ kHz$	$8\ kHz$

Because signals multiplexed over times, hence the frequency is accumulative of all signals frequencies, then the total frequency is 10 + 4 + 4+ 4 = 34 samples/sec.

Number of bits per quantization levels = $\log_2 1024 = \log_2 2^{10} = 10\ bits$.

Bit rate then = $f_s \times N = 34 \times 10^3 \times 10 = 340\ Kbps$

Minimum theoretical bandwidth = 0.5 $f_s\ N = 0.5 \times 340 = 170\ kHz$

Exercise 2.7: A PCM system uses a uniform quantizer followed by a 7 bits binary encoder. The bit rate of the system is $50 \times 10^6\ bits/sec$. What is maximum message BW for which the system operates satisfactorily? What is SNR?

Solution:

Maximum message BW is given as:

$$2\ f_m\ N = 50 \times 10^6 \ \rightarrow \ f_m = \frac{50 \times 10^6}{14} = 3.57\ MHz$$

Signal to noise ratio is given as:

$$SNR = 1.8 + 6N = 1.8 + 6 \times 7 = 43.8\ dB$$

Exercise 2.8: A compact disk record audio signals digitally using PCM. The audio signal BW is 15 kHz. What is Nyquist rate? If $M = 65536$, determine the number of binary digits required to encode a audio signal? Determine the bit rate? Determine the SNR required?

Solution:

The Nyquist rate is:

$$f_s = 2 \times f_m = 30\ kHz = 30000\ \frac{samples}{sec}$$

The number of binary digits is:

$$N = \log_2 65536 = \log_2 2^{16} = 16\ bits$$

SNR required is:

$$SNR = 1.8 + 6N = 1.8 + 6 \times 16 = 97.8\ dB$$

Bit rate of the PCM is given as

$$B_{PCM} = f_s \times N = 30000 \times 16 = 480\ Kbps$$

Exercise 2.9 (homework): design a PCM multiplexer system using a 256 level signal quantizer for transmission of the 3 signals, m_1, m_2 and m_3. Bandlimited to $5\ kHz, 10\ Khz\ and\ 5\ kHz$ respectively. What is the Nyquist rate? Bit rate?