Chapter four

Layout: **10 Hrs.**

1. **High order modulation** (M-ary Modulation or Multi-Level Modulation).
2. QPSK
3. MPSK
4. MFSK
5. MQAM
6. **MATLAB programs.**

Lecture One

Digital Modulation

ASK, PSK & PSK

Digital to Analog Converting (DAC)

Objective of Lecture:

Understand the way by which we convert the bit sequences into analog signals.

Behavioral goals:

- Student will be able to covert bit into analog signal.
- Differentiate between different digital modulations scheme.
- Use appropriate digital modulation technique according to given data

This lecture answer important questions which are:

What is PCM?

Why PCM is important?

How is PCM done?

Where can you exploit PCM?

What are the problems in PCM?

4.6. High Order Modulation (M-ary Modulation or Multi-Level Modulation)

In recent year, we have been trying to obtain more and more communication services out of limited amount of the spectrum as result, channelization in the various system is becoming bandlimited. Sampling process is another issue, where sampling frequency choosing at least two times of signal frequency.

In any attempt to improve this scarcity, high order modulation method has been used which offer greater bandwidth efficiency. The improvement in the bandwidth efficiency come from allocating more bits per signal carrier frequency or greater bits per symbol (level). It clears the advantage of high order modulation can be increased the **bandwidth efficiency** compared to binary digital modulation technique. Disadvantage of M-ary modulations is that they are more complex and more susceptible to noise than compare to binary modulation technique.

In the binary digital modulation number of bits per symbol (carrier) is $n = 1$, it mean number of distinct symbol is $M = 2^1 = 2$ symbols rather than high order modulation, number of bits $n \geq 2$ which mean number of symbols are $M = 2^n$. The relation between number of bits and number of distinct symbols are given as:

$$n = \log_2 M \quad and \quad M = 2^n \tag{1}$$

Transmission speed (data rate or transmission rate) using high order modulation become n times of binary modulation, where transmission speed in bit per second expressed as

$$Transmission\ speed\ in\ bps = n \times\ transmission\ speed\ in\ \frac{symbol}{s} \tag{2}$$

4.6.1. M-ary Phase Shift Keying

The M-ary PSK modulation allocate more than single bit to each carrier symbol uses different phase at each carrier symbol. The difference (or distance) between each carrier symbol is determined by

$$d = \frac{360^o}{M}$$

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \leq f \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the **1101100001** is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000}\ \boldsymbol{symbol/sec}$ using 4PSK digital modulation. Find transmission speed in bps?

- first we determine number of distinct level which is M = 4
- Then we find number of bits per each symbol, $n = \log_2 M \ \rightarrow n = \log_2 2^2 \ \rightarrow n = 2\ bits$.
- Now we find distance between each phase of symbols, which is given as

$$d = \frac{360^o}{M} = \frac{360}{4} = 90^o$$

- We allocate each two bits to distinct symbol as follow:

$$00 \ \rightarrow \ A\sin(2\pi f_c t + \emptyset_0 = 45^0)$$
$$00 \ \rightarrow \ A\sin(2\pi 150{,}000t + \emptyset_0 = 45^0)$$
$$01 \ \rightarrow \ A\sin(2\pi f_c t + \emptyset_1 = 135^0)$$
$$01 \ \rightarrow \ A\sin(2\pi 150{,}000t + \emptyset_1 = 135^0)$$
$$10 \ \rightarrow \ A\sin(2\pi f_c t + \emptyset_2 = 225^0)$$
$$10 \ \rightarrow \ A\sin(2\pi 150{,}000t + \emptyset_2 = 225^0)$$
$$11 \ \rightarrow \ A\sin(2\pi f_c t + \emptyset_3 = 315^0)$$
$$11 \rightarrow \ A\sin(2\pi 150{,}000_c t + \emptyset_3 = 315^0)$$

- Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ symbol$$

- Bit duration in PSK modulation, it given as:

$$T_{symbol} = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

- Transmission speed in bps = $n \times$ transmission speed in $\frac{symbol}{s}$

  Transmission speed in bps = $2 \times\ 50{,}000\ \frac{symbol}{s} = 100{,}000\ bps$

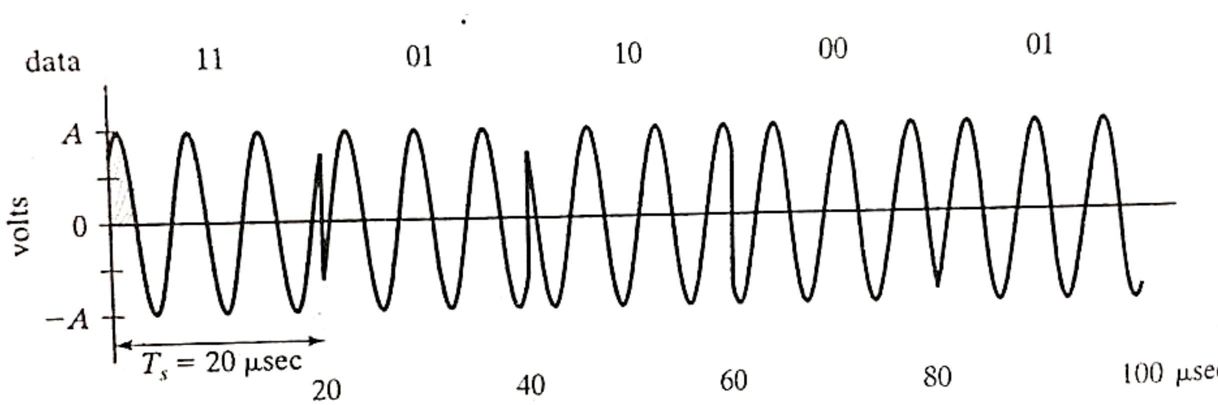


Figure: 4PSK representation

4.6.2. M-ary Frequency Shift Keying

The M-ary FSK modulation allocate more than single bit to each carrier symbol uses different frequency at each carrier symbol. In his case, we can create different symbol with different frequencies. The symbols divided into two groups; first group of symbols (carrier) are frequencies up and second group of symbols are down. Carrier frequencies is given as:

$$up\ frquencies\ \{f_c + \Delta_c, f_c + 2\Delta_c, f_c + 3\Delta_c, f_c + \frac{M}{2}\Delta_c$$

$$down\ frquencies\ \{f_c - \Delta_c, f_c - 2\Delta_c, f_c - 3\Delta_c, f_c - \frac{M}{2}\Delta_c \qquad (3)$$

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $75kHz \leq f \leq 275kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 75 ← **175** → 275). Assume the **1101100001** is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000}\ \boldsymbol{symbol/sec}$ using 4PSK digital modulation, frequency offset $\Delta_c = 25\ kHz$. Find transmission speed in bps?

- first we determine number of distinct level which is M = 4
- Then we find number of bits per each symbol, $n = \log_2 M \ \rightarrow n = \log_2 2^2 \ \rightarrow n = 2\ bits$.
- Carrier frequencies of four groups are given as:
  $up\ frquencies\ \{200000, 225000\}$
  $down\ frquencies\ \{150000, 125000\}$
  $00 \rightarrow A\sin(2\pi(f_c + \ \Delta_c)t)$
  $00 \rightarrow A\sin(2\pi 200000t)$
  $01 \rightarrow A\sin(2\pi(f_c + \ 2\Delta_c) \quad t)$
  $01 \rightarrow A\sin(2\pi 225{,}000t)$
  $10 \rightarrow A\sin(2\pi(f_c - \ \Delta_c)t)$
  $10 \rightarrow A\sin(2\pi 150{,}000t)$
  $11 \rightarrow A\sin(2\pi(f_c - \ 2\Delta_c)t)$
  $11 \rightarrow A\sin(2\pi 125{,}000_c t)$
- Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle\ of\ \mathbf{00} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{200{,}000}{50{,}000} = 4\ cycle\ per\ symbol$$

$$No.of\ Cycle\ of\ \mathbf{01} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{225{,}000}{50{,}000} = 4.5\ cycle\ per\ symbol$$

$$No.of\ Cycle\ of\ \mathbf{10} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ symbol$$

$$No.of\ Cycle\ of\ \mathbf{11} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{125{,}000}{50{,}000} = 2.5\ cycle\ per\ symbol$$

- Bit duration in PSK modulation, it given as:

$$T_{symbol} = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

- Transmission speed in bps = $n \times$ transmission speed in $\frac{symbol}{s}$

Transmission speed in bps = $2 \times\ 50{,}000\ \frac{symbol}{s} = 100{,}000\ bps$

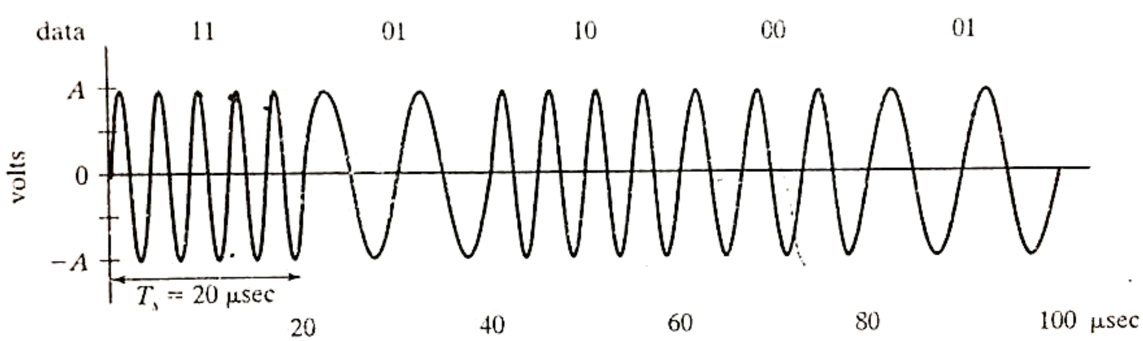


Figure: 4FSK representation

4.6.3. Quaternary Phase Shift Keying (QPSK)

In exiting **4PSK**, we see that each of the four symbols can be decomposed into tow signals: one corresponding to a **Sine (In-phase component)** at carrier frequency and the other corresponding to the **Cosine (Quadrature component)** at given carrier frequency, in such, **4PSK** named as **QPSK**. Mathematically it represented as:

$$S_i\,(t) = A\sin(2\pi f_c t\ +\ \theta_i) \quad for\ i = 0,1,\dots\dots,M$$

$$S_i\,(t) = \underbrace{A\sin(2\pi f_c t) \times \cos(\,\theta_i)}_{In-Phase\ Component} + \underbrace{A\sin(\theta_i) \times \cos(\,2\pi f_c t)}_{Qaudrature\ component}$$

(4)

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \leq f\ \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the **1101100001** is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000}\ \boldsymbol{symbol/sec}$ using 4PSK digital modulation. Find transmission speed in bps?

- first we determine number of distinct level which is M = 4
- Then we find number of bits per each symbol, $n = \log_2 M\ \rightarrow n = \log_2 2^2\ \rightarrow n = 2\ bits.$
- Now we find distance between each phase of symbols, which is given as

$$d = \frac{360^o}{M} = \frac{360}{4} = 90^o$$

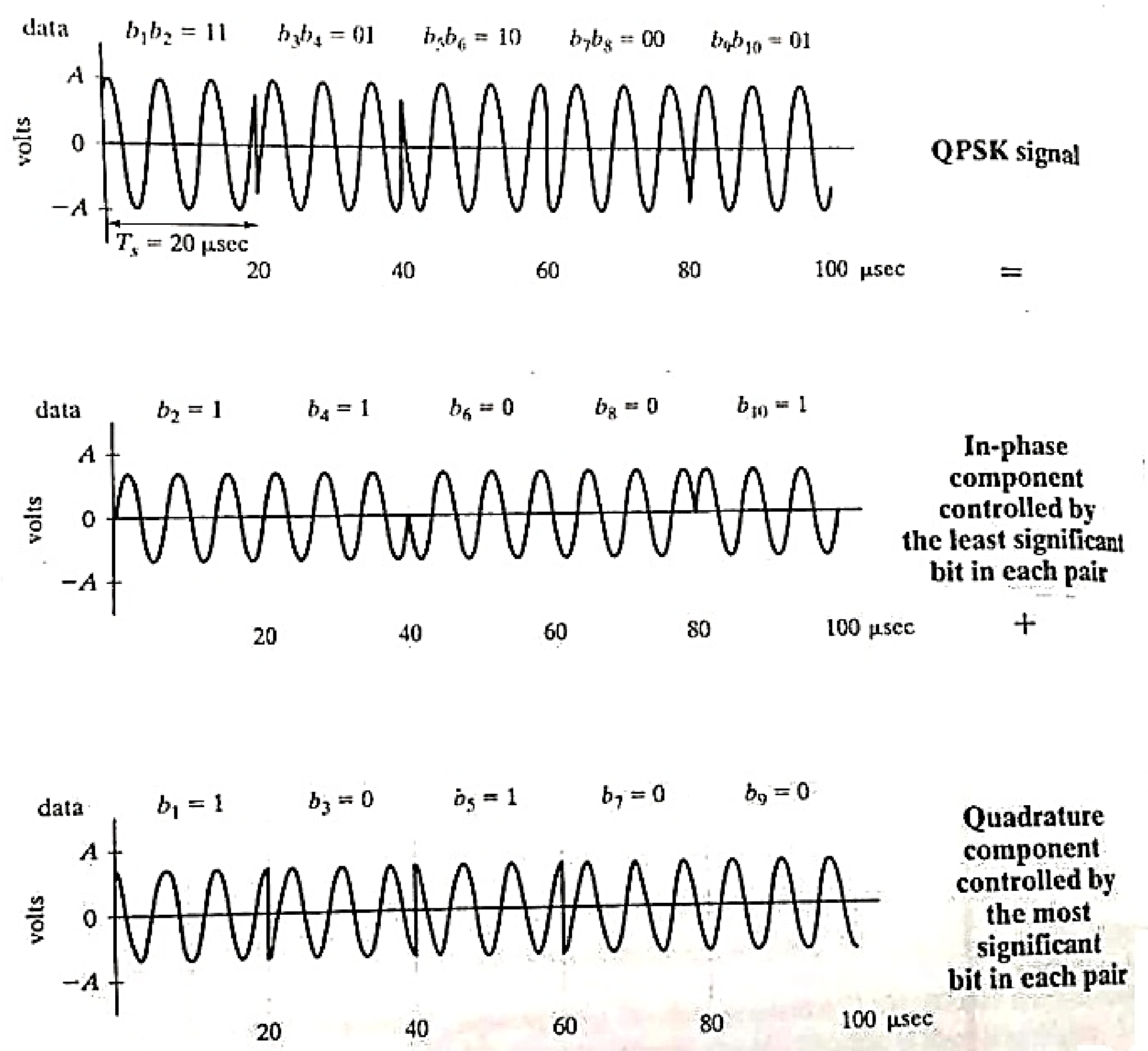
- We allocate each two bits to distinct symbol as follow:

$$00 \rightarrow A\sin(2\pi 150{,}000t +\ 45^0)\ \rightarrow\ \frac{A}{\sqrt{2}}\sin\,(2\pi 150{,}000t) + \frac{A}{\sqrt{2}}\cos(2\pi 150{,}000t)$$

$$01 \rightarrow A\sin(2\pi 150{,}000t +\ 135^0)\ \rightarrow\ \frac{-A}{\sqrt{2}}\sin(2\pi 150{,}000t) + \frac{A}{\sqrt{2}}\cos(2\pi 150{,}000t)$$

$$10 \rightarrow A\sin(2\pi 150{,}000t + 225^0) \rightarrow \frac{-A}{\sqrt{2}}\sin(2\pi 150{,}000t) + \frac{-A}{\sqrt{2}}\cos(2\pi 150{,}000t)$$

$$11 \rightarrow A\sin(2\pi 150{,}000t + 315^0) \rightarrow \frac{-A}{\sqrt{2}}\sin(2\pi 150{,}000t) + \frac{-A}{\sqrt{2}}\cos(2\pi 150{,}000t)$$



- Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ symbol$$

- Bit duration in PSK modulation, it given as:

$$T_{symbol} = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

- Transmission speed in bps = $n \times$ transmission speed in $\frac{symbol}{s}$

  Transmission speed in bps = $2 \times\ 50{,}000\ \frac{symbol}{s} = 100{,}000\ bps$

4.6.3. M-ary Quadrature Amplitude Modulation (MQAM)

We know that carrier can have three parameters which are variable to represent different kind of modulation. These parameters are Amplitude which produce ASK modulation, Phase which produce PSK modulation and Frequency which produce FSK modulation.

So far, we have developed digital modulation Bandpass modulation technique that vary only one parameters of the carrier signal (amplitude, frequency and phase). The question come to our mind, is it possible to **vary two parameters** at same time**? Of course yes**, with multi-parameters approach modulation system can provide good performance from receiver point of view.

One of the most popular multi-parameters modulation is **Quadrature Amplitude Modulation (QAM)**. In the QAM, we use Amplitude and Phase to represent each pair of bits. QAM mathematically represented as:

$$S_i(t) = A\ a_i \cos(2\pi f_c t)\ - A\ b_i \sin(2\pi f_c t)\quad for\quad i = 0,1, \ldots\ldots, M \tag{5}$$

In which, a_i is amplitude of the In-phase component, b_i is the amplitude of the Quadrature component. The coefficient $a_i\ and\ b_i$ are given as:

$$(a_i, b_i) = \begin{pmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots (L-3, L-1) & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots (L-3, L-1) & (L-1, L-3) \\ \vdots & \vdots & \vdots & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots (L-3, -L+1) & (L-1, -L+1) \end{pmatrix}$$

In fact, 4QAM is same as 4PSK, therefore we consider 8QAM and 16QAM.

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \le f \le 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the **110 111 000 101100 010 011 001** → is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000\ symbol/sec}$ using **8QAM** digital modulation. Find transmission speed in bps**?**

- =
- Then we find number of bits per each symbol, $n = \log_2 M \rightarrow n = \log_2 2^3 \rightarrow n = 3\ bits.$
- Now we find distance between each phase of symbols, which is given as

$$d_{phase} = \frac{360^o}{M} = \frac{360}{8} = 45^o$$

- Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ symbol$$

- Bit duration in PSK modulation, it given as:

$$T_{symbol} = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

- Transmission speed in bps = $n \times$ transmission speed in $\frac{symbol}{s}$

Transmission speed in bps = $3 \times\ 50{,}000\ \frac{symbol}{s} = 1500{,}000\ bps$

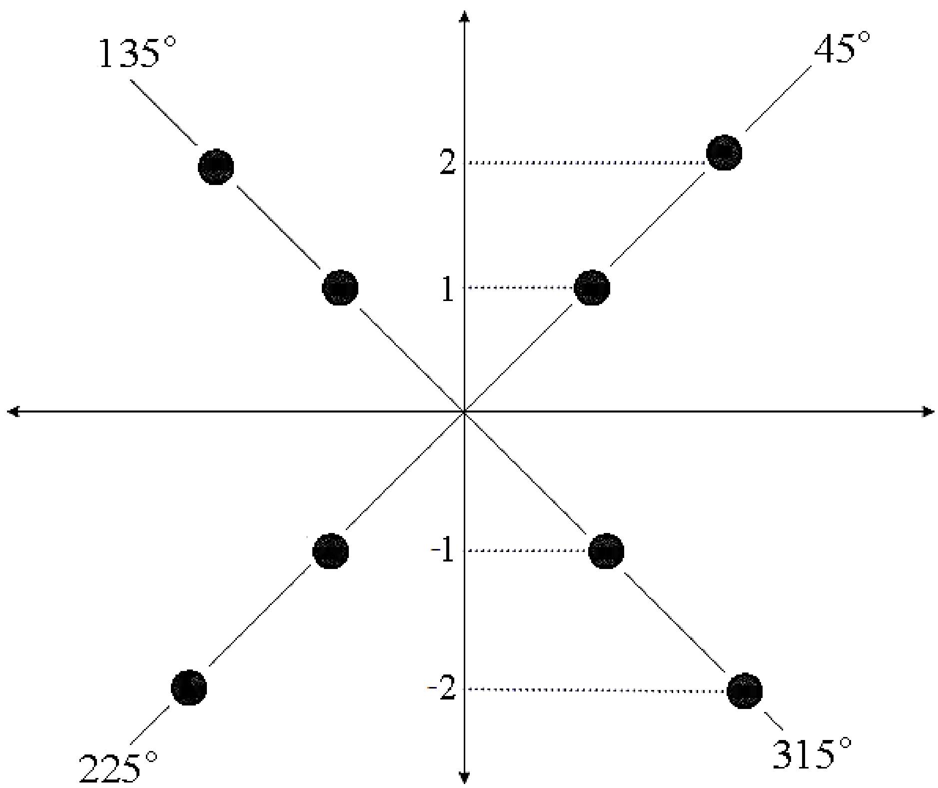


Figure: 8QAM Constellation.

$\mathbf{000} \rightarrow \quad \mathbf{1}\sin(2\pi f_c t) + \mathbf{1}\cos(2\pi f_c t) \ \rightarrow \quad \mathbf{1}\sin(2\pi 150000 t) + \mathbf{1}\cos(2\pi 150000 t)$

$\mathbf{001} \rightarrow \quad \mathbf{2}\sin(2\pi f_c t) + \mathbf{1}\cos(2\pi f_c t) \ \rightarrow \quad \mathbf{2}\sin(2\pi 150000 t) + \mathbf{1}\cos(2\pi 150000 t)$

$\mathbf{010} \rightarrow -\mathbf{1}\sin(2\pi f_c t) + \mathbf{1}\cos(2\pi f_c t) \ \rightarrow -\mathbf{1}\sin(2\pi 150000 t) + 1\cos(2\pi 150000 t)$

$\mathbf{011} \rightarrow -\mathbf{2}\sin(2\pi f_c t) + \mathbf{1}\cos(2\pi f_c t) \ \rightarrow -\mathbf{2}\sin(2\pi 150000 t) + \mathbf{1}\cos(2\pi 150000 t)$

$\mathbf{100} \rightarrow -\mathbf{1}\sin(2\pi f_c t) - \mathbf{1}\cos(2\pi f_c t) \ \rightarrow -\mathbf{1}\sin(2\pi 150000 t) - \mathbf{1}\cos(2\pi 150000 t)$

$\mathbf{101} \rightarrow -\mathbf{2}\sin(2\pi f_c t) - \mathbf{2}\cos(2\pi f_c t) \ \rightarrow -\mathbf{2}\sin(2\pi 150000 t) - \mathbf{2}\cos(2\pi 150000 t)$

$\mathbf{110} \rightarrow \quad \mathbf{1}\sin(2\pi f_c t) - \mathbf{1}\cos(2\pi f_c t) \ \rightarrow \quad \mathbf{1}\sin(2\pi 150000 t) - \mathbf{1}\cos(2\pi 150000 t)$

$\mathbf{111} \rightarrow -\mathbf{2}\sin(2\pi f_c t) + \mathbf{2}\cos(2\pi f_c t) \ \rightarrow -\mathbf{2}\sin(2\pi 150000 t) + \mathbf{2}\cos(2\pi 150000 t)$

HW: suppose a channel can be pass frequencies in range $50kHz \leq f \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the **0000 0001 0010 0011 0100 0101 0110 0111 1000 1001 1010 1011 1100 1101 1110 1111** → is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000\ symbol/sec}$ using **16QAM** digital modulation. Find transmission speed in bps**?**