Chapter Two

Layout: **10 Hrs.**

1. Introduction.
2. Pulse Code Modulation (PCM).
3. Differential Pulse Code Modulation (DPCM).
4. Delta modulation.
5. Adaptive delta modulation.
6. Sigma Delta Modulation (SDM).
7. Linear Predictive Coder (LPC).
8. **MATLAB programs.**

Lecture Three

Introduction to Waveforms Encoding

And

Sampling Theorem

Layout:

1. Waveform Encoding
2. Sampling Theorem

Objective of Lecture:

Understand waveform encoding techniques and sampling theorem algorithms; it is problems, Mathematical expression and numerical examples

Behavioral goals:

The student will be able to convert the signal from continues time to discrete time signal using sampling theorem and student could have knowledge about different types of analog to digital conversion technique.

This lecture answer important questions which are:

What is waveform encoding mean?

Why waveform encoding is needed or important?

What are the types of waveform encoding modulation?

How the is sampling theorem?

What the sampling theorem do for signal?

Waveform Encoding or Signal encoding is technique by which signal converted from the analog to binary form (code)

2.1. Introduction

In *continuous-wave (CW) modulation – analog modulation*, some parameter of a sinusoidal carrier wave is varied continuously in accordance with the message signal. This is in direct contrast (وهذا معاكس تماما) to pulse modulation, which we study in the present chapter. In *pulse modulation*, some parameter of a *pulse train* is varied in accordance with the message signal. In this context, we may distinguish two families of pulse modulation (sometimes named as analog to digital conversion):

1. Analog - Pulse Modulation - APM (such as Pulse amplitude modulation PAM)
2. Digital Pulse – (Code) Modulation - DPM (such as: pulse code modulation PCM)

In analog pulse modulation, a periodic pulse train is used as the carrier wave, and some characteristic feature of each pulse (e.g., amplitude, duration, or position) is varied in a continuous manner (different amplitude range in accordance to original analog signal) in accordance with the corresponding *sample* value of the message signal. Thus, in analog pulse modulation, information is transmitted basically in pulses digital form, but the transmission takes place at continues times. In digital pulse-code modulation, on the other hand, the message signal is represented in a form that is *discrete in both time and amplitude*, thereby permitting its transmission in digital form as a sequence of *coded pulses*. Simply put, digital pulse modulation has *no* CW counterpart.

We wonder which is important in digital communication systems, APM or DPM. In fact, digital pulse to code modulation is important, because most of digital communication technique is done over bits form such as channel coding, encryption, spread coding and source coding. But all the previous technique cannot be done over digital pulses, see Fig 1.4 in the **lecture 1**.

'Any natural signal is in analog form'. Therefore, to meet the basic requirement of any type of digital signal processing and digital communication is essential and prior step to convert the electrical form (through transducer) of the natural analog signal into digital form, because digital modulator or any type of digital signal processor does not accept analog signal as its input. The process by which the signal converted to binary form is call signal encoding. In analog domain, the signal that is of concern is continuous in both time and amplitude. The process of discretization of the analog signal in both time domain and amplitude levels yields the equivalent digital signal. The conversion of analog signal to bit form is done by a three step process**:**

1. Discretization in time – **Sampling**
2. Discretization of amplitude levels – **Quantization**
3. Converting the quantized **-** samples to binary form using **Coding/Encoding**

In fact, there are different technique of signal (waveform) encoding:

1. Pulse Code Modulation
2. Differential pulse code modulation (**DPCM**).
3. delta-modulation (**DM**)
4. Sigma-delta-modulation (**SDM**)
5. Adaptive delta modulation (**ADM**)
6. Linear predictive coder (**LPC**)

Note: *Analog pulse-modulation systems rely on the sampling process to maintain continuous amplitude representation of the message signal. In contrast, digital pulse-modulation systems use not only the sampling process but also the quantization process, which is non-reversible. Quantization provides a representation of the message signal that is discrete in both time and amplitude. In so doing, digital pulse modulation makes it possible to exploit the full power of digital signal-processing techniques.*

2.2. Sampling Theorem

First and foremost, in digital communication, it is required to transform a continues-time signal into discrete-time signal. This conversion from continuous to discrete time is done by process called *sampling*. Through use of the sampling process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.

Sampling Theorem: suppose signal $g(t)$ is *strictly band-limited (i.e., signal with frequency component from zero to W Hz)*, with no frequency components higher than *W Hz*. That is, the Fourier transform $G(f)$ of the signal $g(t)$ has the property that is zero for $|f| > W$. Where the signal can uniquely determine and reconstructed from the sampled signal if sampled frequency twice the original signal frequency, i.e. $f_S \geq 2W$, Or $f_S \geq NOS * W$, where NOS number of samples.

Where, T_S called sampling period $1/2W$ and f_S is sampling rate or number of sample per second $f_S = 2W$. The sampling rate $f_S = 2W$ is called the Nyquist rate and corresponding time interval $T_S \frac{1}{2W}$ is called Nyquist interval. Nyquist rate is the conditions under which a signal can be exactly reconstructed from its samples.

The problems in sampling theorem: the processing of sample in the time domain results in *periodic spectrum in frequency domain* with period equal to sampling frequency f_S. Therefore, overlapping between periodic spectrums may appear after sampling, such problem named as aliasing. Hence, Nyquist rate prevent such problem to appear after sampling, see Fig. 2.3. This answer the question, why aliasing appear after sampling.

To illustrate the aliasing effect, let consider signal with frequency $f_m = 4$ KHz, then required sampling rate is $f_S = 8\ KHz$, but if the sampling frequency used is 5 KHz, the f_m frequency higher than $\frac{f_S}{2}$ ($\frac{f_S}{2}$ is called folding frequency), therefore, 1 KHz reflected inside the f_m after reconstruction of analog signal, see Fig.2.4. Hence 1 KHz will appear inside the original signal in addition to 4 KHz frequency after LPF limited to 4 KHz bandwidth.

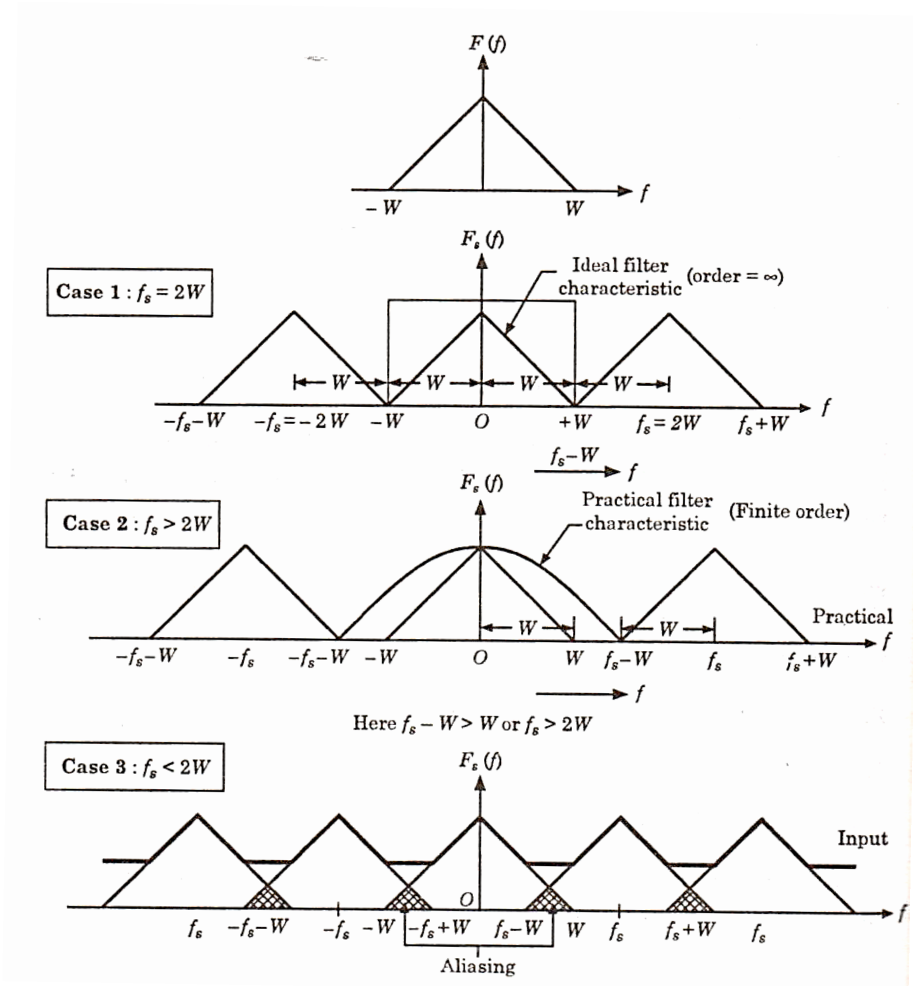

Figure 2.3. Nyquist rate and aliasing problem description.

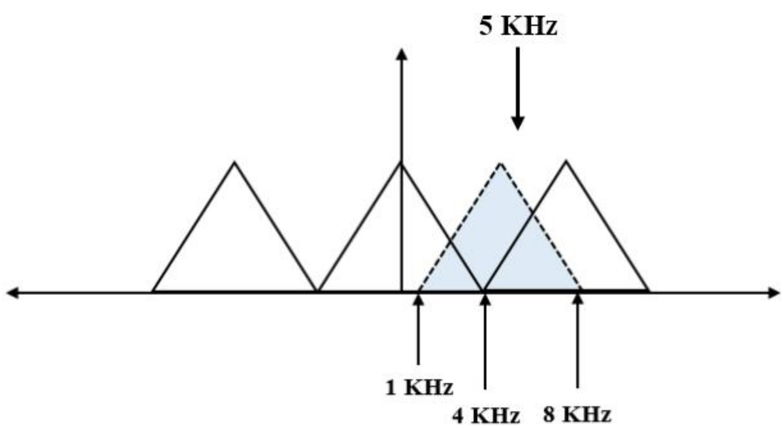

Figure 2.4. Frequency Folding description

There are three sampling method that can be employed:

1. Ideal or instantaneous sampling
2. Natural sampling
3. Flat topped sampling

In this study, **ideal sampling** is considered. Ideal sampling method is accomplished as follow (Mathematical Expression): Let consider continues time signal *g(t)* apply to sampler given an out *g(nT_S)* see Fig. 2.6, where T_S is sampled period after uniformly sampling *g(t)* in time, the sampler output is given as

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s) \tag{1}$$

We refer $g_\delta(t)$ to as the *instantaneously (ideal) sampled signal*. The $\delta(t - nT_S)$ term represents a delta function positioned at time $t = nT_{S.}\ n = 1, 2, 3, \dots, N$, N is number of samples. From the definition of the delta function which has unit area see Fig 2.5. where:

$$\delta(t) = 1 \qquad t = 0$$
$$\delta(t) = 0 \qquad elsewhere$$

and

$$\delta(t - T_S) = 1 \qquad t = T_S$$
$$\delta(t) = 0 \qquad elsewhere$$

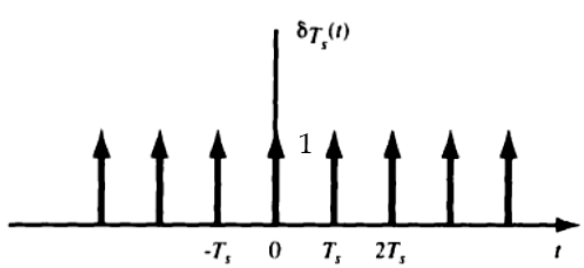

Figure 2.5. Dirac delta function $\delta(nT_S)$.

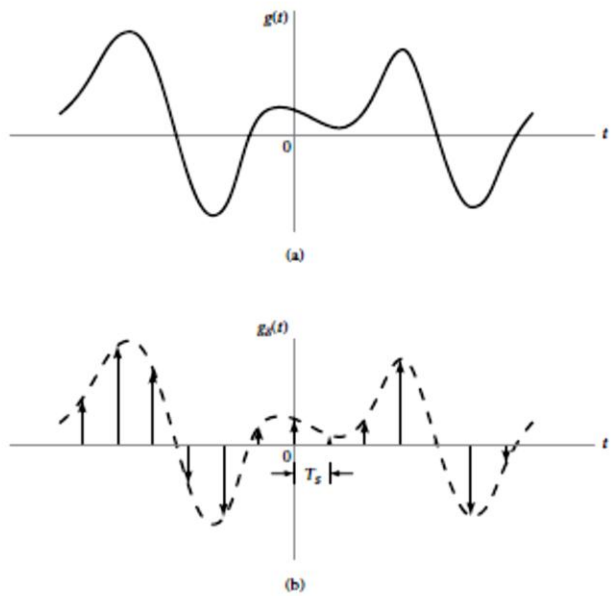

Figure 2.6 Sampling process. (a) Analog signal $g(t)$ (b) Instantaneously sampled representation of $g(t)$

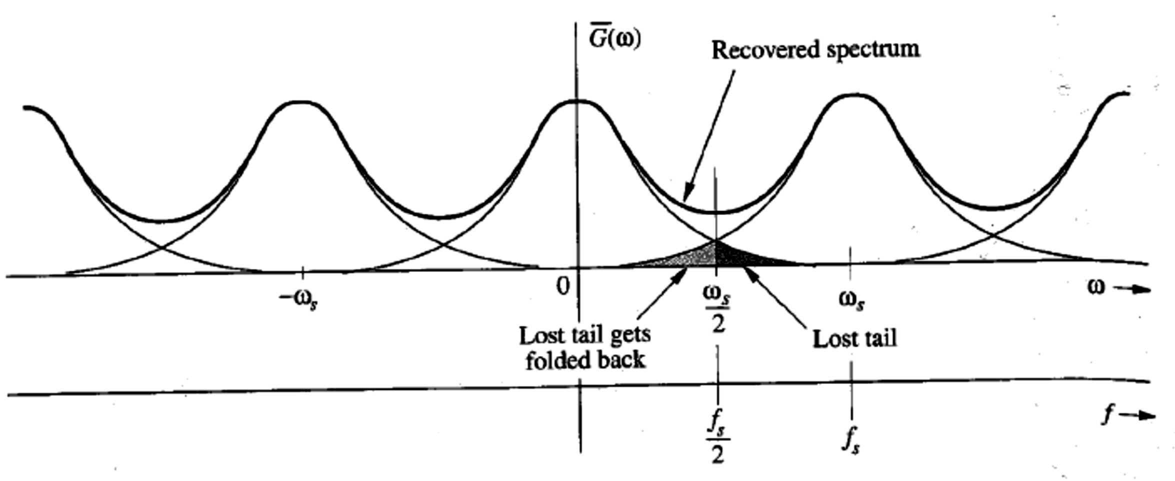

Figure 2.7 High Frequency Tail

To combat the effect of aliasing, the following steps are considers:

1. Prior to sampling (before sampling process), a low pass pre-aliasing or anti-aliasing filter is used to attenuate high frequency components of the signal. In fact, not high frequency will appear in original frequency, but the lost tail gets back or folded back to the original frequency. See figure 2.7.

2. Sampling the signal at rate higher than Nyquist rate. The using of higher sampling rate simplifies the design of reconstruction filter to recover the original signal from it samples. But higher sampling frequency also indicates more samples, which implies more storage space or more memory requirements and more complexity in sampler design.

Note: the ideal samples with zero width (i.e. zero area base) are physically non-existent, actually samples train are pulses with finite width. See figure 2.8.

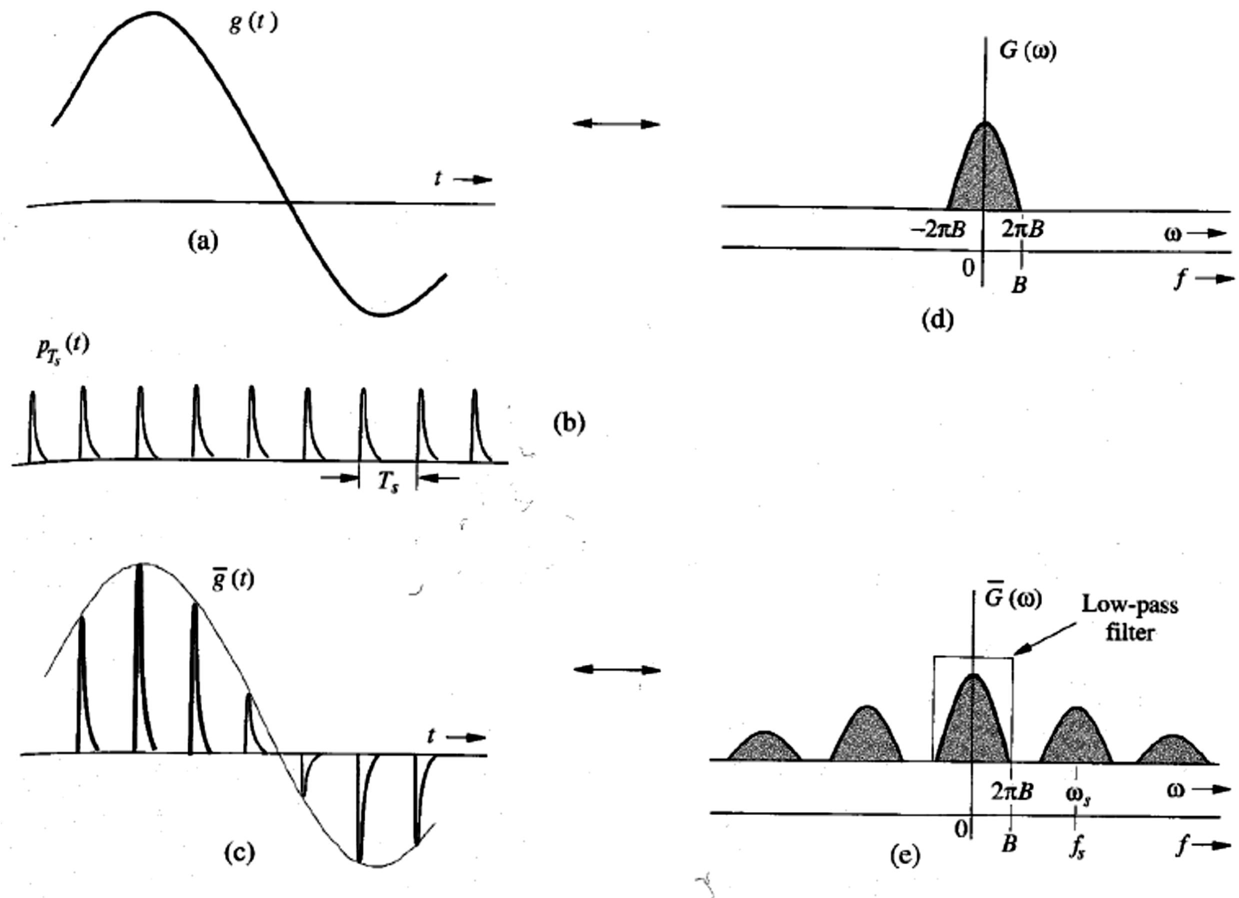

Figure 2.8: practical sampling and it is Fourier transform

Exercise 2.1: Find the Nyquist rate and Nyquist interval for each of the following signals:

(a) $\boldsymbol{m(t) = 3\cos(100\pi t) + 10\sin(400\pi t)\cos(\pi t)}$

(b) $\boldsymbol{m(t) = 5\cos(1000\pi t)\cos(4000\pi t)}$

(c) $\boldsymbol{m(t) = {}^{\sin(200\,\pi\,t)}/_{\pi t}}$

(d) $\boldsymbol{m(t) = \left({}^{\sin(200\,\pi\,t)}/_{\pi t}\right)^2}$

Solution:

(a) $m(t) = 3\cos(100\,\pi\,t) + 10\sin(\,400\,\pi\,t\,)\cos(\,\pi\,t)$

$100\,\pi\,t = \omega_1 t = 2\,\pi\,f_1\,t$, f_1 = 50 Hz

$400\,\pi\,t = \omega_2 t = 2\,\pi\,f_2\,t$, f_2 = 200 Hz

Since maximum frequency present in *m(t)* in f_2 = 200 Hz,

Hence, Nyquist rate = f_s= 2 $f_m = 2 \times 200 > 400\,Hz$.

Nyquist interval = T_s = ${}^1/_{f_s} = {}^1/_{400} = 2.5\,ms$

(b) $m(t) = 5\cos(1000\,\pi\,t)\cos(\,4000\,\pi\,t) = 2.5\,(\,\cos(5000\,\pi\,t) + \cos(\,3000\,\pi\,t))$

$5000\,\pi\,t = \omega_1 t = 2\,\pi\,f_1\,t$, f_1 = 2500 Hz

$4000\,\pi\,t = \omega_2 t = 2\,\pi\,f_2\,t$, f_2 = 2000 Hz

Since maximum frequency present in *m(t)* in f_2 = 2500 Hz,

Hence, Nyquist rate = f_s= 2 $f_m = 2 \times 2500 > 5000\,Hz$.

Nyquist interval = T_s = ${}^1/_{f_s} = {}^1/_{5000} = 200\,\mu s$

(c) $m(t) = {}^{\sin(200\,\pi\,t)}/_{\pi t}$

$200\,\pi\,t = \omega_1 t = 2\,\pi\,f_1\,t$, f_1 = 100 Hz

Since maximum frequency present in *m(t)* in f_2 = 100 Hz,

Hence, Nyquist rate = f_s = 2 $f_m = 2 \times 100 > 200\,Hz$.

Nyquist interval = T_s = ${}^1/_{f_s} = {}^1/_{200} = 5\,ms$

(d) $\boldsymbol{m(t) = \left({}^{\sin(200\,\pi\,t)}/_{\pi t}\right)^2 = \frac{1}{2(\pi t)^2}\,(1 + \cos(\,400\,\pi t))}$

$400\,\pi\,t = \omega_1 t = 2\,\pi\,f_1\,t$, f_1 = 200 Hz

Since maximum frequency present in *m(t)* in f_2 = 200 Hz,

Hence, Nyquist rate = $f_s = 2\, f_m = 2 \times 200 > 400\ Hz$.

Nyquist interval = T_s = $1/_{f_s} = 1/_{400}$ = 2.5 ms

Exercise 2.2: Find the Nyquist rate, if the Nyquist interval $T_s = 125\,\mu s$ for the following signals, then draw the sampled signal? What is the number of samples if signal end at $t = 4\ ms$? Find the signal frequency? Write mathematical expression?

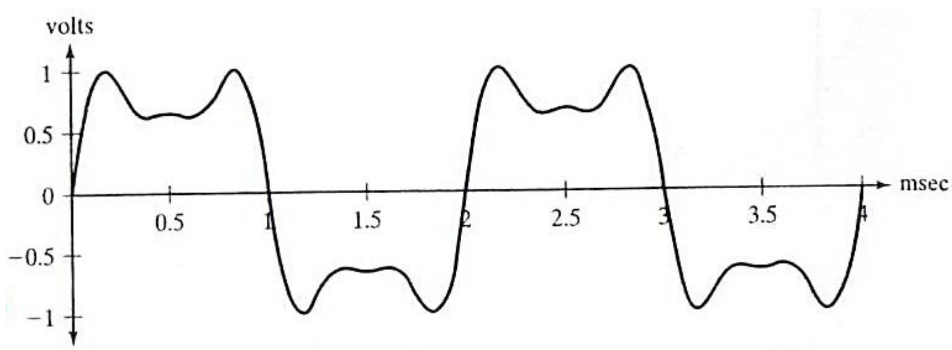

Figure 2.5. Exercise 2.2.

Solution:

Nyquist rate is $f_s = 1/T_s = 1000000/125$ = 8 $samples/sec$

Number of samples = t/T_s = $4/125\ 000$ = 32 samples

Signal frequency = $f_m = 1000/4 = 250\ Hz\ (cycle/sec)$ for single period, signal given Fig. 2.2 is repeated itself after 2 ms , hence, $f_m = 500\ Hz$.

Sampled signal at sample interval 125 μs.

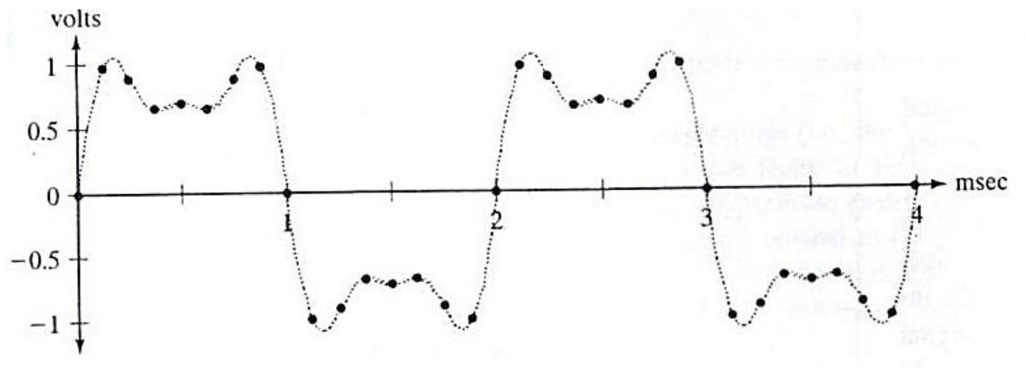

Figure 2.6. Exercise 2.2.

Mathematical expression of sampling:

$$g_{\delta}(t) = \sum_{n=0}^{32} g(nT_s)\, \delta(t - nT_s)$$