Chapter Two

Layout: **10 Hrs.**

1. Introduction.
2. Pulse Code Modulation (PCM).
3. Differential Pulse Code Modulation (DPCM).
4. Delta modulation.
5. Adaptive delta modulation.
6. Sigma Delta Modulation (SDM).
7. Linear Predictive Coder (LPC).
8. **MATLAB programs.**

Lecture Five

Differential Pulse Code Modulation

Objective of Lecture:

Understand the way by which we convert the analog signal to binary bits.

Behavioral goals:

The student will be able to use new analog to bit form conversion and understand how it possible to reduce bit rate (and improving bandwidth efficiency) by keeping the quality of signal after reconstruction.

This lecture answer important questions which are:

What is DPCM?

Why DPCM is important?

How is DPCM done?

Where can you exploit DPCM?

What are the problems in the DPCM?

2.5. Differential Pulse Code Modulation (DPCM)

For yet another form of digital pulse modulation, we recognize that when a voice or video signal is sampled at a rate slightly higher than the Nyquist rate, the resulting sampled signal is found to show a high degree of correlation between adjacent samples. The meaning of this high correlation is that, in an average sense, the signal does not change rapidly from one sample to the next.

When these highly correlated samples are encoded as in a standard PCM system, the resulting encoded signal contains *redundant information*. Redundancy means that codewords are not absolutely essential to the transmission of information which are generated as a result of the encoding process. By removing this redundancy before encoding, we obtain a more *efficient bits rate or bandwidth transmission*, compared to PCM (i.e.B_{PCM}).

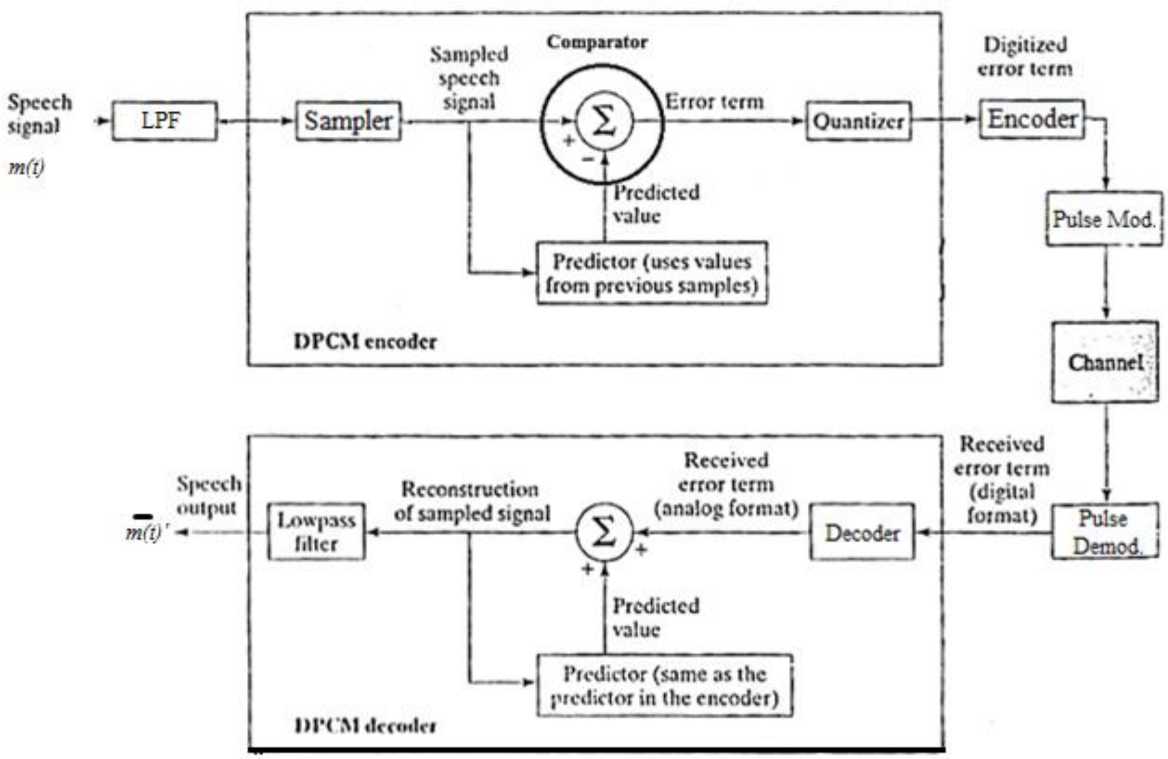

Figure 2.15 DPCM Transmitter and Receiver.

DPCM is based on linear predication. In particular, if we know the past behavior of a signal up to a certain point in time, it is possible to make some inference about its future values; such a process is commonly called *prediction*. In fact, a good prediction lead to better performance. The linear predictor function is given as:

$$\hat{m}(nT_s) = \sum_{i=1}^{K} a_i \, m(nT_s - iT_s) \tag{1}$$

In which, $\hat{m}(nT_s)$ is the predicted value n_{th} sample, a_i is predictor coefficients, $m(nT_s - iT_s)$ is n_{th} past samples, and K is predictor order. For more details about linear predictor see appendix.

<u>The DPCM transmitter is describe as follow:</u>

1. First the signal passed through pre-sampled filter.
2. The analog signal is sampled at Nyquist rate or over sampled at sampling rate f_s, in which the signal converted from continues-time signal $m(t)$ to discrete-time signal $m(nT_s)$.
3. The sampled signal amplitude $m(nT_s)$ move to comparator and predictor (predictor is linear predictor can be implemented using tapped-delay line or FIR filter), predictor estimate the current sampled value $\hat{m}(nT_s)$ based on amplitude of past sampled values $m(nT_s)$.
4. The predicted sampled signal $\hat{m}(nT_s)$ move to comparator to produce error term $e(nT_s) = m(nT_s) - \hat{m}(nT_s)$, where $e(nT_s) \ll m(nT_s)$.
5. The error term moved to quantizer which is round-off the value either up or down, and
6. Then each quantization level encoded with $N\ bits$, which is given as $N = \log_2(M)$.
7. The $M \times N$ bits converted from parallel to serial to enter the pulse modulation which converted the bit to electrical pulse.

8. Electrical pulse is transmitted over physical channel (we already know channel consideration is physical).

At the receiver, the DPCM decoder uses same method as the encoder to predict the value of the present sample. After the error term is received and demodulated, it is added to prediction (i.e., $\widehat{m}(nT_s) - m(nT_s) = e(nT_s) \rightarrow \widehat{m}(nT_s) + e(nT_s) = m(nT_s)$), producing again the actual amplitude of present sample.

If prediction algorithm is chosen properly, the error term will have lower dynamic range than original signal (i.e., $e(nT_s) \ll m(nT_s)$) and thus need fewer quantizing bits to achieve the much less quantization error which make DPCM require lower rate than PCM. DPCM require less quantization level because it depend on error term that is represented with less dynamic range. Error term depend on the predictor performance, if the signal well predicted, the error term reduces and dynamic range become less which result less quantization level (i.e. M).

2.5.1. SNR of Differential Pulse Code Modulation

SNR is important metric in communication system which measure the systems performance, let examine DPCM SNR. In fact, of the PCM in dB is given as:

$$(SNR)_q = 1.8 + 6.02\ N \tag{2}$$

The SNR of DPCM is given by:

$$(SNR)_q = 1.8 + 6.02\ N_{DPCM} \tag{3}$$

The number of Bits of DPCM is given as:

$$N_{DPCM} = \log_2 \frac{e(nT_s)}{q_{e_{PCM}}} \tag{4}$$

Finally, The SNR of DPCM is given by:

$$(SNR)_q = 1.8 + 6.02\ \log_2 \frac{e(nT_s)}{q_{e_{PCM}}} \tag{5}$$

2.5.2. Transmission Bandwidth (Bit Rate) of DPCM

A signal $m(t)$ bandlimited to W Hz, in the sequel, sampling rate f_s required is $2W\ sample/sec$, if each quantized samples encoded to $N\ bits$, then total channel bandwidth required is given as

$$B_{DPCM}\ =\ 2\ N_{DPCM} \times W\ = 2\log_2(M) \times W =\ bits/sec \tag{6}$$

Yields,

$$B_{DPCM}\ =\ N_{DPCM}\ f_s\ =\ bits/sec \tag{7}$$

From (7), minimum bandwidth required for PCM is proportional to the message signal bandwidth and number of bit per quantization level or step-size. $N_{DPCM}\ < N_{PCM}$ due to dynamic range of DPCM signal is less than dynamic range of PCM signal (i.e., $e(nT_s) \ll m(nT_s)$). Where, N_{DPCM} in term of quantization error of PCM is given as (founded by Dr.Ahmed Alkhayyat):

$$N_{PDCM} = \log_2\ \frac{e(nT_s)}{q_{e\ \ PCM}} \tag{8}$$

Exercise 2.10: suppose we want to convert the analog signal shown in Fig. 2.16 into digital format. If the sampling rate $8000\ samples/sec$ and using uniform quantization at $8\ bits/sample$. Find number of levels, Nyquist interval, number of samples, quantization error PCM, linear predictor function, and DPCM transmitter parameters then show how DPCM is more efficient than PCM in bit rate term? $e(nT_s) = 0.5\ v$.

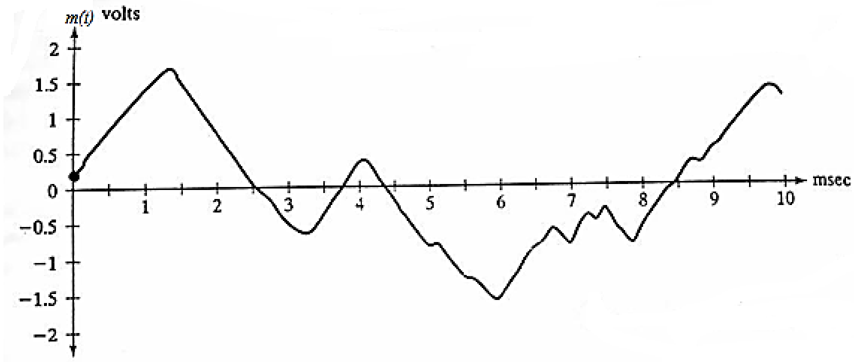

Figure 2.16: original signal before DPCM

Solution:

Hence number of quantization level is

$$M = 2^N = 2^8 = 256\ levels$$

The Nyquist interval is

$$T_s = \frac{1}{f_s} = \frac{1}{8000} = 0.125\ msec$$

Number of samples using $T_s = 0.125\ ms$ is

$$\text{Number of Samples } = \frac{time\ end\ of\ signal}{sampling\ interval} = \frac{10}{0.125} = 80\,\frac{samples}{signal}$$

The maximum quantization error of PCM is given as

$$q_e = \pm\frac{dynamic\ range\ of\ signal}{2^{N+1}} = \pm\frac{4}{2^9} = \pm 0.0078125\ volts$$

DPCM TRANSMITTER				
n	T_s $(msec)$	Predicted values $\widehat{m}(nT_s) = 0.75\, m(n-1) + 0.20\, m(n-2) + 0.05\, m(n-3)$	Actual values $m(nTs)$	Error Term $e(nT_s)$
0	0	$\widehat{m}(0T_s) = 0.75\ (0) + 0.20\ (0) + 0.05(0) = 0v$	0.23	0.23
1	0.125	$\widehat{m}(1T_s) = 0.75\ (0.23) + 0.20\ (0) + 0.05(0) = 0.172v$	0.38	0.2075
2	0.25	$\widehat{m}(2T_s) = 0.75\ (0.38) + 0.20\ (0.23) + 0.05(0) = 0.33v$	0.56	0.229
3	0.375	$\widehat{m}(3T_s) = 0.75\ (0.56) + 0.20\ (0.38) + 0.05(0.23) = 0.5v$	0.73	0.2225
4	0.5	$\widehat{m}(4T_s) = 0.75\ (0.73) + 0.20\ (0.56) + 0.05(0.38) = 0.678v$	0.9	0.2215
5	0.625	$\widehat{m}(5T_s) = 0.75\ (0.9) + 0.20\ (0.73) + 0.05(0.56) = 0.847v$	1.05	0.201
6	0.7	$\widehat{m}(6T_s) = 0.75\ (1.05) + 0.20\ (0.9) + 0.05(0.73) = 1.004v$	1.2	0.196
7	0.875	$\widehat{m}(7T_s) = 0.75\ (1.2) + 0.20\ (1.05) + 0.05(0.9) = 1.155v$	1.35	0.195
8	1	$\widehat{m}(8T_s) = 0.75\ (1.35) + 0.20\ (1.2) + 0.05(1.05) = 1.305v$	1.48	0.175
9	1.125	$\widehat{m}(9T_s) = 0.75\ (1.48) + 0.20\ (1.35) + 0.05(1.2) = 1.44v$	1.62	0.18
etc.	etc.	etc.	etc.	etc.

Suppose that linear predictor with order $K = 3$, hence the predictor function is:

$$\widehat{m}(nT_s) = \sum_{i=1}^{3} a_i\, m(n-i)$$

The linear predictor function after evaluating coefficients, a_1 , a_2 and a_3 , is written as:

$$\widehat{m}(nT_s) = 0.75\, m(n-1) + 0.20\, m(n-2) + 0.05\, m(n-3)$$

The amplitude of each sample after sampling process is given as:

$$m(nTs) = \{0.23, 0.38, 0.56, 0.73, 0.9, 1.05, 1.2, 1.35, \ldots.\}$$

$$m(nTs) = \{0T_s, 1T_s, 2T_s, 3T_s, 4T_s, 5T_s, 6T_s, 7T_s, \ldots.\}$$

$$n = \{0,1,2,3,4,5,6,7, \ldots.\}$$

DPCM transmitter is shown in the Table below which consist, nT_s, predicted value $\widehat{m}(n)$, actual value $m(nTs)$, error term $e(nT_s)$. Because the dynamic range of DPCM signal depend on the error term, hence it is smallest that original signal. Where maximum dynamic range of DPCM is $\pm$ 0.5, by which we can estimate the bit rate of DPCM as:

$$q_e = \pm\frac{dynamic\ range\ of\ signal}{2^{N+1}} = \pm\frac{0.5}{2^{N+1}} = \pm 0.0078125\ volts$$

$$2^{N+1} = 64 \rightarrow \log_2 2^{N+1} = \log_2 64 \rightarrow N + 1 = 6 \rightarrow N = 5\ bits\ per\ level$$

Hence number of bits reduced per level from 8 bits to 5 bits and gives same quantization error. The DPCM bit rate efficiency is given as:

$$B_{PCM} = N\ f_s = 8000 \times 8 = 64\ kpbs$$
$$B_{DPCM} = N\ f_s = 8000 \times 5 = 40\ kpbs$$
$$bandwidth\ efficiency = \frac{40}{64} = 62.5\ \%$$

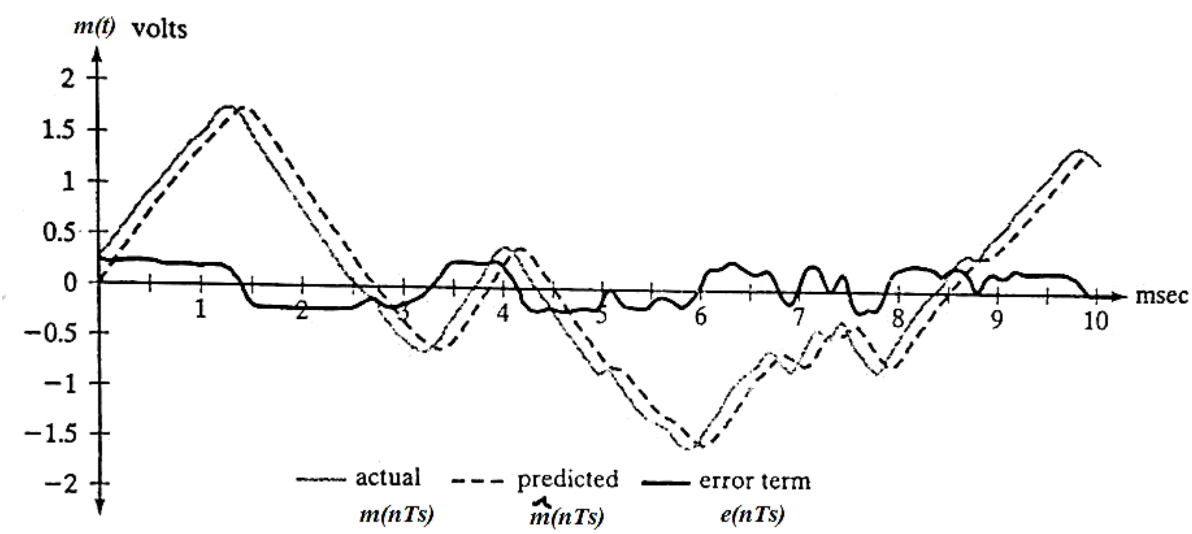

Figure2.17: Original (actual), predicated and error term signal