Chapter Two

Layout: **10 Hrs.**

1. Introduction.
2. Pulse Code Modulation (PCM).
3. Differential Pulse Code Modulation (DPCM).
4. Delta modulation.
5. Adaptive delta modulation.
6. Sigma Delta Modulation (SDM).
7. Linear Predictive Coder (LPC).
8. **MATLAB programs.**

Lecture Five

Delta Modulation

Objective of Lecture:

Understand the way by which we convert the analog signal to binary bits.

Behavioral goals:

This lecture answer important questions which are:

What is DM?

Why DM is important?

How is DM done?

Where can you exploit DM?

2.6. Delta Modulation (DM)

In fact, the voice information quality is not primary importance, hence it is possible to reduce the data rate in such the voice could be recognizable at receiver side. In order to reduce bit rate, we need to design code modulation with less quantization levels or less number of bits for each level, in such case, we may use another digital pulse modulation technique known as delta modulation.

2.6.1. Transmitter and Receiver of DM

In *delta modulation* (DM), an incoming message signal is oversampled (i.e., at a rate much higher than the Nyquist rate) to purposely increase the *correlation* between adjacent samples of the signal. The increased correlation is done so as to allow the use of a simple quantizing (round-off) strategy for constructing the encoded signal.
In its basic form, DM provides a staircase approximation to the oversampled version of the message signal. Unlike PCM, the difference between the input signal and its approximation is quantized into only two levels (only zero and one digit, that is mean number of bits N is 2 bits)—namely, $\pm\Delta$, corresponding to positive and negative *differences.*
Where, $-\Delta$ indicate 1 and $+\Delta$ indicate 0. In other word, 1 signify a positive error term and 0 signify a negative error term.
Thus, if the predicted samples falls below the input signal at any sampling epoch, it is increased by $+\Delta$, the error term is positive, hence we signify it with 1. If, on the other hand, the approximation lies above the signal, it is diminished (reduced) by $+\Delta$, the error term is negative, hence we signify it with 0. The DM block diagram is given in Fig. 2.16

$$\hat{m}(nT_s) = \begin{cases} \hat{m}(nT_s - 1) + \Delta & if \quad \hat{m}(nT_s - 1) \le m(nT_s - 1) \\ \hat{m}(nT_s - 1) - \Delta & if \quad \hat{m}(nT_s - 1) > m(nT_s - 1) \end{cases} \tag{1}$$

The principle of DM is resemble to (mimics مشابه) guessing game, suppose we ask you to guess a number between 1 and 10, after you guess, we tell that your guess was too low. Then, for your next guess you will increase your last guess by some increment. Because

we deal with error term, so if error is large, we reduce it by $+\Delta$, and if the error is negative, we increase by $+\Delta$.

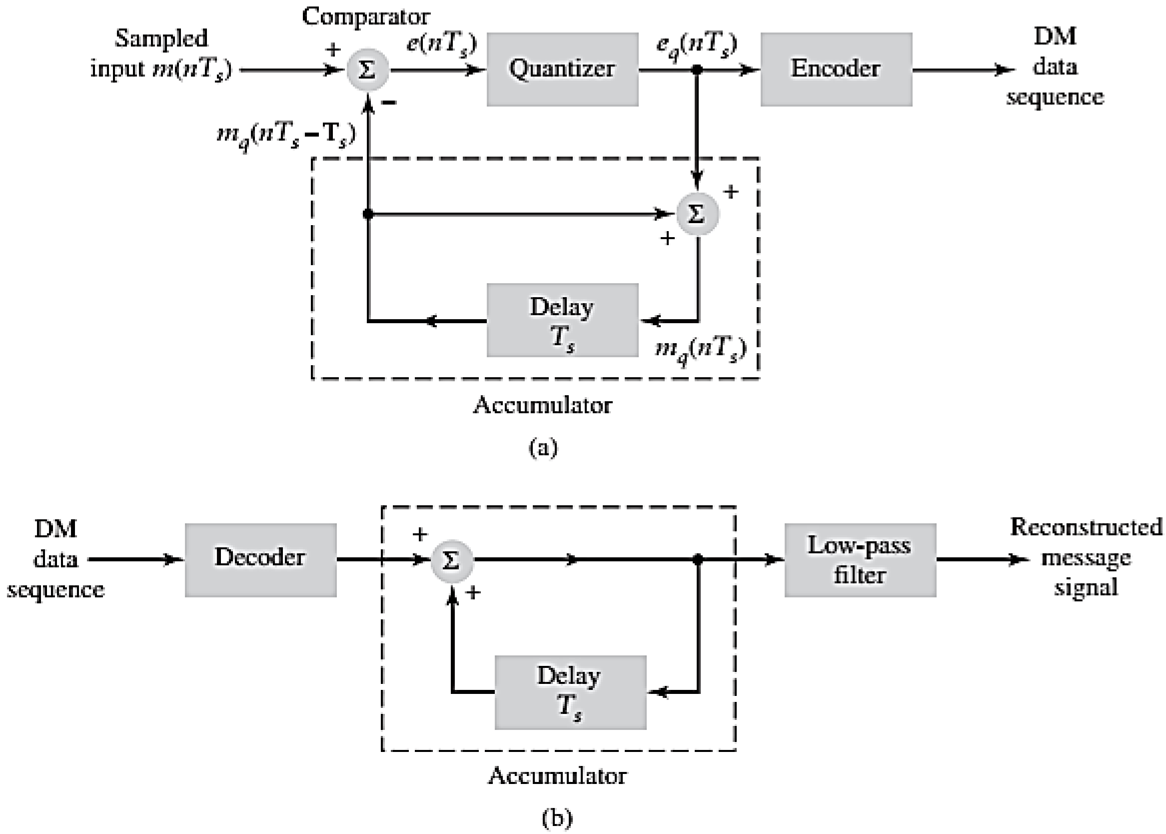


Figure 2.16 DM system: (*a*) Transmitter and (*b*) receiver.

Let denote $m(t)$ is original signal, then $m(nT_s)$ is sampled signal. The basic principle of delta modulation may then be formalized in the following set of three discrete-time relations:

$$e(nT_s) = m(nT_s) - m_q(nT_s - T_s) \tag{2}$$

$$e_q(nT_s) = \Delta \; sgn\,(e(nT_s)) \tag{3}$$

Where T_s is the sampling period; $e(nT_s)$ is an *error signal* representing the difference between the present sample $m(nT_s)$ value of the input signal and output of the accumulator $m_q(nT_s - T_s)$, and $e_q\,(nTs)$ is the quantized version of $e(nT_s)$ and sgn[.] is the signum

function, assuming the value +1 or -1. The quantizer output is finally encoded to produce the desired DM data.

2.6.2. Transmission Bandwidth (Bit Rate) of DM

A signal $m(t)$ bandlimited to $W\ (f_m)$ Hz, in the sequel, sampling rate f_s required is $2W\ sample/sec$, if each quantized samples encoded to $N\ bits$, then total channel bandwidth required is given as

$$B_{DM} = 2\,N \times W\ bits/sec \tag{4}$$

Delta modulation is signal bit allocation, i.e., $N_{DM} = 2\ bit$, therefore minimum bandwidth required is given as

$$B_{DM} = 2f_s = bits/sec \tag{5}$$

2.6.3. SNR of Delta Modulation

The signal to quantization noise ratio is written as

$$SNR = \frac{E\ \{m(t)^2\}}{E\ \{e_q\ (t)^2\}} \tag{6}$$

In which, $E(.)$ is average notation. Making the assumption that the quantization noise in DM is uniformly distributed over $\pm\Delta$, the mean-square quantization error power is

$$E\left\{e_q(t)^2\right\} = \int_{-\Delta}^{+\Delta} \frac{1}{2\Delta}\ e^2\ de \ = \frac{1}{2\Delta}\frac{e^3}{3} = \frac{\Delta^2}{3} \tag{7}$$

We assume that this power is spread evenly over all frequencies up to the sampling frequency $2f_s$, then we re-write (30) as

$$E\left\{e_q(t)^2\right\} = \frac{\Delta^2}{3} \times \frac{1}{2f_s} = \frac{\Delta^2}{6f_s}$$

(8)

However, there is still the lowpass filter in the DM receiver — if the cutoff frequency is set to the maximum frequency f_m, finally we write normalized noise power as:

$$E\left\{e_q(t)^2\right\} = \int_{-f_m}^{+f_m} \frac{\Delta^2}{6f_s}\ df = \frac{\Delta^2 f_m}{3f_s}$$

(9)

Suppose $m(t) = A\sin(2\pi\, f_m)$, then average power signal is given as:

$$E\left\{m(t)^2\right\} = 0.5\, A^2$$

(10)

To avoid overload slop, the maximum amplitude is given as

$$A = \frac{\Delta\, f_s}{2\pi f_m}$$

Finally, SNR of DM is given as:

$$SNR_{DM} = \frac{A^2}{2} \times \frac{3f_s}{\Delta^2\, f_m} = \frac{\Delta^2 f_s^2}{8\,\pi^2\, f_m} \times \frac{3}{\Delta^2}\frac{f_s}{f_m} = \frac{3}{8\,\pi^2} \times \frac{f_s^3}{f_m^3}$$

(11)

It is clear that, SNR_{DM} of DM is directly proportional to f_s^3 , we can conclude that the SNR_{DM} improve rapidly by increasing the sampling rate which make DM performance better and easy to detect.

In fact, Delta modulation systems are subject to two types of quantization error:

1. Slope Overload Distortions2.
2. Granular Noise

2.6.3.1. Overload Slop and Granular Problem in DM

The quantized signal $m_q(t)$ needs to closely follow the original signal of $m(t)$ in order to the recovered quantized signal resembles $m(t)$, see Fig 2.17. Taking a careful look at the signal shows situations where quantized signal is unable to follow the original one as the slope of original signal is higher than that of quantized signal.

Therefore, overload slop problem can be define as divergence between original signal slop and the quantized signal because the quantized process (quantizer steps) cannot follow the original signal therefore the received quantized signal will be difficult to recover.

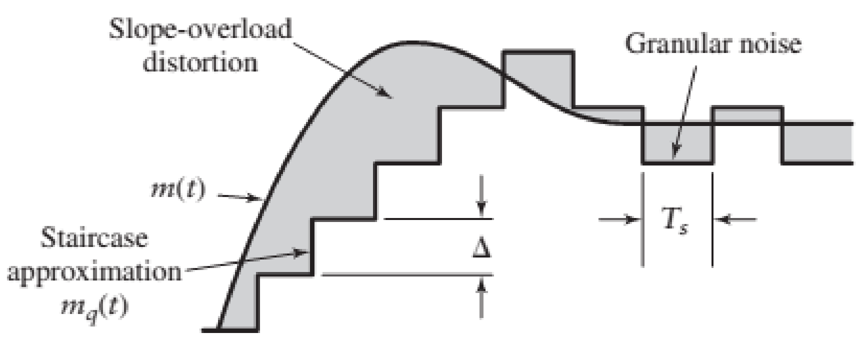


Figure 2.17. Slope-overload distortion and granular noise, in delta modulation.

To avoid slop-overload, the condition is

$$\frac{d}{dt}\, m(t) \;\le \Delta\, f_s \tag{12}$$

If the original signal is $m(t) = A\sin(2\,\pi\, f_m\, t)$, then

$$\frac{d}{dt}\, m(t) \;= A\,(2\,\pi\, f_m)\;\sin(2\,\pi\, f_m)$$

$$\left|\frac{d}{dt}\, m(t)\right|_{max} = A\; 2\pi f_m$$

Apply the condition of (35), we obtain

$$A\; 2\pi f_m \;\le \Delta\, f_s$$

$$A \;\; \le \frac{\Delta\, f_s}{2\pi f_m}$$

This is to say that, the maximum amplitude of the input signal should be less than or equal to $\frac{\Delta f_s}{2\pi f_m}$ to avoid slop over load.

Also it possible to conclude that optimum step size for sine wave signal is given as

$$\Delta_{optimum} = \frac{A\ 2\pi f_m}{f_s}$$

Granular noise, In contrast to slope-overload distortion, *granular noise* occurs when the step size is too large relative to the local slope characteristic of the original message signal $m(t)$.

Exercise 2.11: delta modulation system**,** the message signal is $m(t) = 0.1 \sin(2\pi\ 10^{-3} t)$, and $f_s = 20\ KHz$, does slop overload occur at step size $\Delta = 60\ mV$ or $\Delta = 4\ mV$?

Solution:

$$\left| \frac{d}{dt}\ m(t) \right|_{max} = 0.1 \times 2\pi \times 10^{-3} = 200\pi$$

$$200\pi\ \leq \Delta f_s$$

$$\Delta \geq \frac{200\pi}{f_s} \rightarrow \frac{200\ \pi}{20\ 000} \rightarrow \Delta = 31.4\ mV$$

The optimum step size is $\Delta > 31.4\ mV$ to avoid slop overload error. Therefore, slop overload does not occur at step size 60 mV, while slop overload occur at step size 4 mV.

Exercise 2.12: Find sine wave signal amplitude for minimum slop over load error in DM system. If the step size is 1V with sampling interval (repetition signal) T_s = 1ms and message signal frequency is 100Hz.

Solution:

The maximum signal amplitude is given by

$$A \le \frac{\Delta f_s}{2\pi f_m} \le \frac{1 \times 1000}{2 \times \pi \times f_m} \le \frac{1000}{2\pi 100} \le 1.59\,V$$

$$A = 1.59\,V \text{ is maximum amplitude}$$

Exercise 2.13: signal to be transmitted using DM, where message signal is given as

$$m(t) = 10\cos 1000\pi t + 5\cos 1500\pi t$$

1. Determine an appropriate f_s and step size for the DM.
2. Determine SNR for DM system.

Solution:

1. The appropriate f_s is given as

$$f_s = 2\,f_m$$

Where, f_m is obtain from the message signal as

$$2f_m\,\pi t = 1500\pi t \quad \rightarrow f_m = \frac{1500}{2} = 750\,Hz$$

Then simply we can find sampling frequency (Nyquist rate) as

$$f_s = 2 \times 750 = 1.5\,kHz$$

Nyquist rate for DM system should be several time larger than Nyquist rate condition, i.e., 10 times of normal value, hence f_s is given as

$$f_{s(DM)} = 10\,f_s = 10 \times 1.5 = 15\,kHz$$

The appropriate step-size is obtain as

$$\left|\frac{d}{dt}\,m(t)\right|_{max} \le \Delta\,f_s$$

$\frac{d}{dt}\,m(t)$ is obtain as

$$\left|\frac{d}{dt}\,m(t)\right|_{max} = 10 \times 1000\pi + 5 \times 1500\pi = 17500\pi$$

Then, step size is obtain as

$$17500\,\pi \le \Delta\,15000 \;\rightarrow \Delta = \frac{17500\,\pi}{15000} = 3.67\,V$$

2. SNR of DM is obtain as

$$SNR_{DM} = \frac{3}{8\,\pi^2} \times \frac{f_{s(DM)}^3}{f_m^3} = \frac{3}{8\,\pi^2} \times \frac{15000^3}{750^3} = 304.27$$

$$SNR_{DM} = 10\log_{10} 304.27 = 24.8\,dB$$

Question: is it possible to improve signal to noise ratio without increasing transmitted power to detect the received signal easily?

The answer is YES, SNR increase through increasing sampling rate which increase the correlation between samples that improve signal reconstruction at receiver. Therefore, let make sampling frequency 100 times more of message frequency, as result we write f_s as

$$f_s = 20\,f_m = 100 \times 750\,Hz = 75\frac{samples}{sec}$$

Therefore, SNR_{DM} is written as

$$SNR_{DM} = 10\log_{10}\frac{3}{8\,\pi^2} \times \frac{f_{s(DM)}^3}{f_m^3} = 10\log_{10}\frac{3}{8\,\pi^2} \times \frac{75000^3}{750^3} = 45.8\,dB$$

Question: is it possible to increase sampling rate to infinity?

The answer NO, because increasing sampling rate required sophisticated devices which increase the device complexity and cost.

Exercise 2.14: if 25 dB SNR of DM systems, determine the quantized bit rate for analog signal with bandwidth (f_m) 3400 kHz?

Solution:

We need SNR in linear form instead of logarithmic form

$$SNR_{DM} = 10^{\frac{SNR}{10}} = 10^{\frac{25}{10}} = 316.22$$

Then we apply SNR of DM as

$$SNR_{DM} = \frac{3}{8\,\pi^2} \times \frac{f_{s(DM)}^3}{f_m^3} \rightarrow 316.22 = 0.04 \times \frac{f_{s(DM)}^3}{3400} \rightarrow \frac{316.22 \times (3400)^3}{0.04} = f_{s(DM)}^3$$

$$f_{s(DM)}^3 = 310 \times 10^{14} \rightarrow f_{s(DM)} = 67{,}729$$

Due to that DM is single bit allocation, hence the bit rate transmission of DM is given as

$$B_{DM} = f_{s(DM)} = 67\ kbps$$

Exercise 2.15: The input signal is $m(t) = 0.01\,t$. The modulator operates at sampling frequency of $20Hz$ and has step size $2\ mV$. Sketch the delta modulator output?

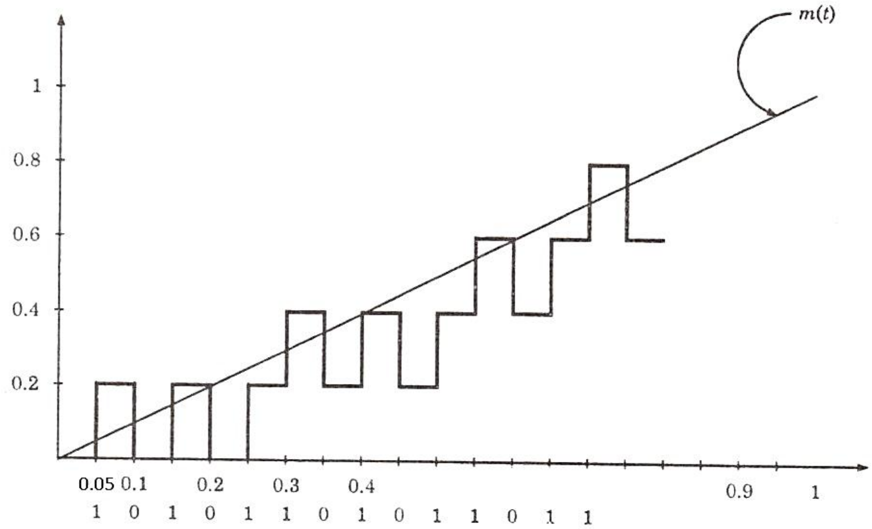
Solution:

Nyquist interval is obtain as

$$T_s = \frac{1}{f_s} = \frac{1}{20} = 0.05$$

We need to find number of samples, which is obtain as:

$$number\ of\ samples = \frac{t}{T_s} = \frac{1}{0.05} = 20\ samples/sec$$



Let describe how delta modulation work is:

1. At $T_s = 0$, always $m(t) > 0$, allocate 1 digit, next $+\Delta$ (added to next step)
2. At $T_s = 0.05$, the $m(t) < +\Delta$, $m(t)$ is low, allocate 0 digit, next $-\Delta$
3. At $T_s = 0.1$, the $m(t) > 0$, $m(t)$ is high, allocate 1 digit, next $+\Delta$
4. At $T_s = 0.15$, the $m(t) < +\Delta$, $m(t)$ is low, allocate 0 digit, next $-\Delta$
5. At $T_s = 0.2$, the $m(t) > 0$, $m(t)$ is high, allocate 1 digit, next $+\Delta$
6. At $T_s = 0.25$, the $m(t) > \Delta$, $m(t)$ is high, allocate 1 digit, next $+\Delta$
7. At $T_s = 0.3$, the $m(t) < 2\Delta$, $m(t)$ is low, allocate 0 digit, next $-\Delta$
8. At $T_s = 0.35$, the $m(t) > \Delta$, $m(t)$ is high, allocate 1 digit, next $+\Delta$
9. At $T_s = 0.4$, the $m(t) < 2\Delta$, $m(t)$ is low, allocate 0 digit, next $-\Delta$
10. At $T_s = 0.45$, the $m(t) > \Delta$, $m(t)$ is high, allocate 1 digit, next $+\Delta$
11. At $T_s = 0.5$, the $m(t) < 2\Delta$, $m(t)$ is low, allocate 0 digit, next $+\Delta$
12. At $T_s = 0.55$, the $m(t) > 3\Delta$, $m(t)$ is high, allocate 1 digit, next $-\Delta$
13. At $T_s = 0.6$, the $m(t) < 2\Delta$, $m(t)$ is low, allocate 0 digit, next $+\Delta$
14. At $T_s = 0.65$, the $m(t) < 3\Delta$, $m(t)$ is low, allocate 0 digit, next $+\Delta$
15. At $T_s = 0.7$, the $m(t) > 4\Delta$, $m(t)$ is high, allocate 1 digit, next $-\Delta$

Exercise 2.16: use DM to covert signal given bellow into binary form, then find sampling frequency, message frequency and SNR_{DM}. The optimum step size $\Delta = 6.25\ mV$.

Solution:

From the figure bellow, frequency message is

$$f_m = \frac{1}{t} = \frac{1}{0.5 \times 10^{-3}} = 2000Hz$$

The sampling frequency of DM is given as

$$f_s = 5 \times f_m = 5 \times 2000 = 10{,}000\ Hz$$

The SNR of DM is given as

$$SNR_{DM} = \frac{3}{8\,\pi^2} \times \frac{f_{s(DM)}^3}{f_m^3} \rightarrow 0.04\ \times \frac{10{,}000\text{^}3}{2000\text{^}3}\ = 5$$

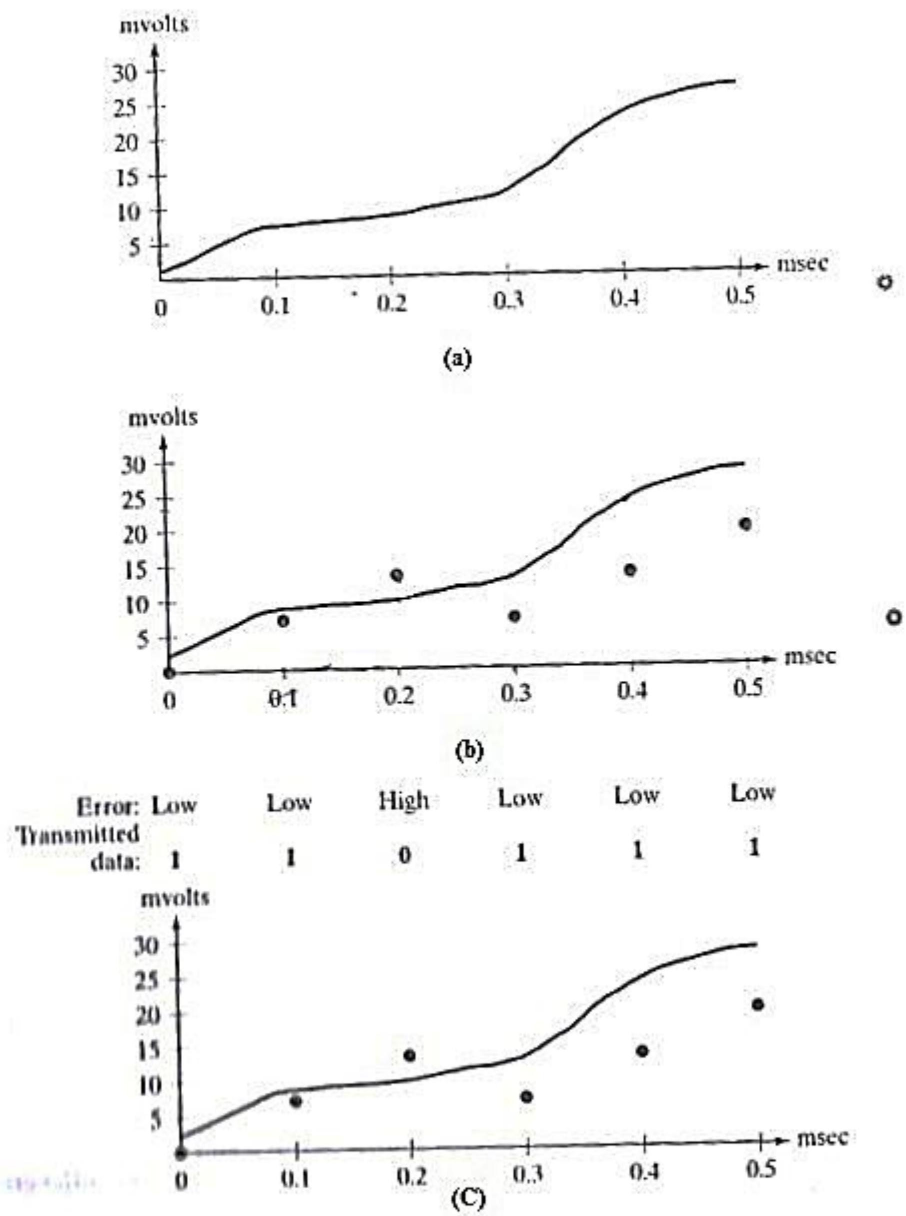


Figure 2.18. DM Process, (a) message signal, (b) quantized signal, (c) binary representation