Lecture Two

Signal Classifications

Layout:

1. General Definition of signal.
2. Classification of signals.

Objective of Lecture:

Understand general definition signal types in communication systems.

Behavioral goals:

Make student differentiate between signal types and the suitable processing type of each signal.

1.1. General Definition of signal

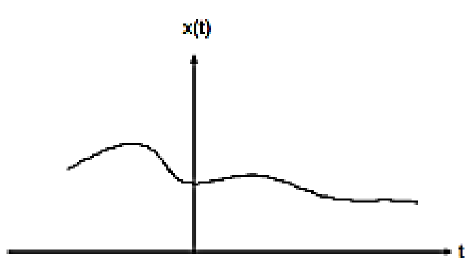
A signal is function that convey information about systems or attributes of some phenomena. Signal is not necessarily an electrical quantities, may it physical quantities such as sound wave, light wave, and earthquake wave. However, to perform activities such as synthesizing (producing توليد الاشارة), transporting (نقل الاشارة), recording, analyzing and modifying (معالجة الاشارة) the signal, *it is often utilize the signal in electrical quantity*. We will consider **signal** to be some function of an independent variable such as time.

Alternatively, signal is the physical quantity by which information is carried between at least two individual's devices (بين جهازين). In other words, signal by which physical quantity communication is done.

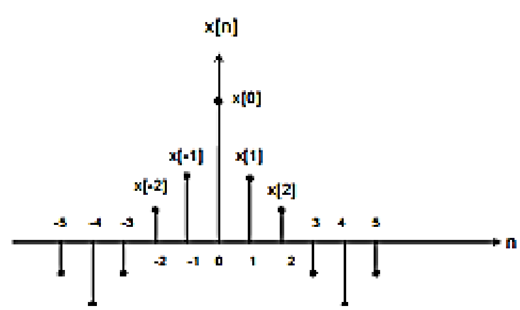
1.2. Classification of signals

1. Continues **time** and discrete **time** signals (continuous and discrete in time)
2. Analog and digital signals (continuous and discrete in amplitude)
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and probabilistic signals

1.2.1. Continues and discrete time signals



Graphical representation of continuous-time signal.



Graphical representation of discrete-time signal.

Figure 1.1. Continues time and discrete time signal representation

An analog signal $x(t)$ is a continuous function of time that shown in Fig 1.1. $x(t)$ is defined for all t (e.g. speech signal), by comparison a discrete signal $x(n)$ is one that exists only at discrete times, it is characterized by a sequence of number for each $x(kn)$.

Exercise: give continues and discrete time signals example**?**

Continues time signal:

$$x(t) = \sin(2\pi f t) \qquad t \in R^{+}$$

$$x(t) = t^2 + t + 1 \qquad t \in R^{+}$$

$$x(t) = \begin{cases} 1 & for\ t \geq 0 \\ 0 & otherwise \end{cases}$$

$$x(t) = \begin{cases} 1 & for\ t \geq 0 \\ 3 & for\ t < 0 \end{cases}$$

Note: continues time signal defined for all value of t

Discrete time signals:

$$x(t) = \begin{cases} 1 & for\ t = 1, 2, 3, 5 \\ 0 & otherwise \end{cases}$$

$$x(t) = \begin{cases} 1 & for\ t = 2,3,4 \\ 3 & for\ t = 0,6,7 \end{cases}$$

Note: discrete time signal not defined for all value of t

1.2.2. Analog and digital signal

The concept of analog are confused with that of continuous time, the two are not same. The same are true for concept of discrete time and digital signal, where discrete time can be

digital but the inverse are not true. A signal whose amplitude can take any value in a continuous range (different values) is an analog signal. This means that an analog signal amplitude can take on an infinite number of values. A digital signal can take only a finite number of values for t. For a signal to be define as digital, the number of values need not be restricted to two (It can take any finite number as in the case M-ary signal). *The terms continuous time and discrete time define the nature of signal along the time (horizontal axis). The terms digital and analog define the nature of the signal amplitude (vertical axis)*. Fig (1.2) shows examples of various types of signals.

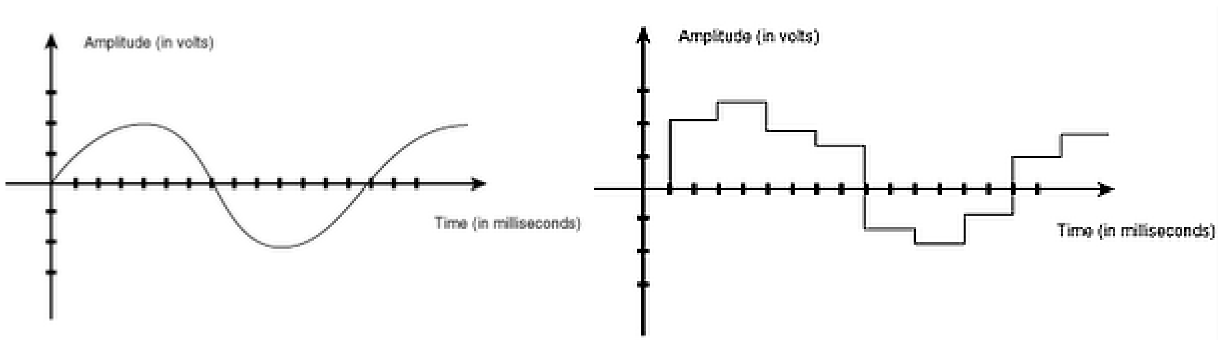


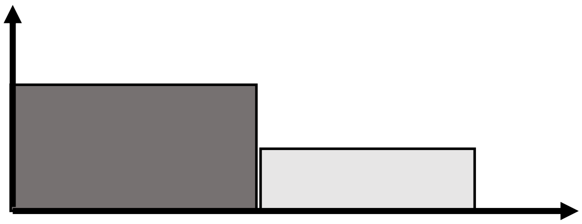
Figure 1.2. Analog and digital signals.

Analog signal:

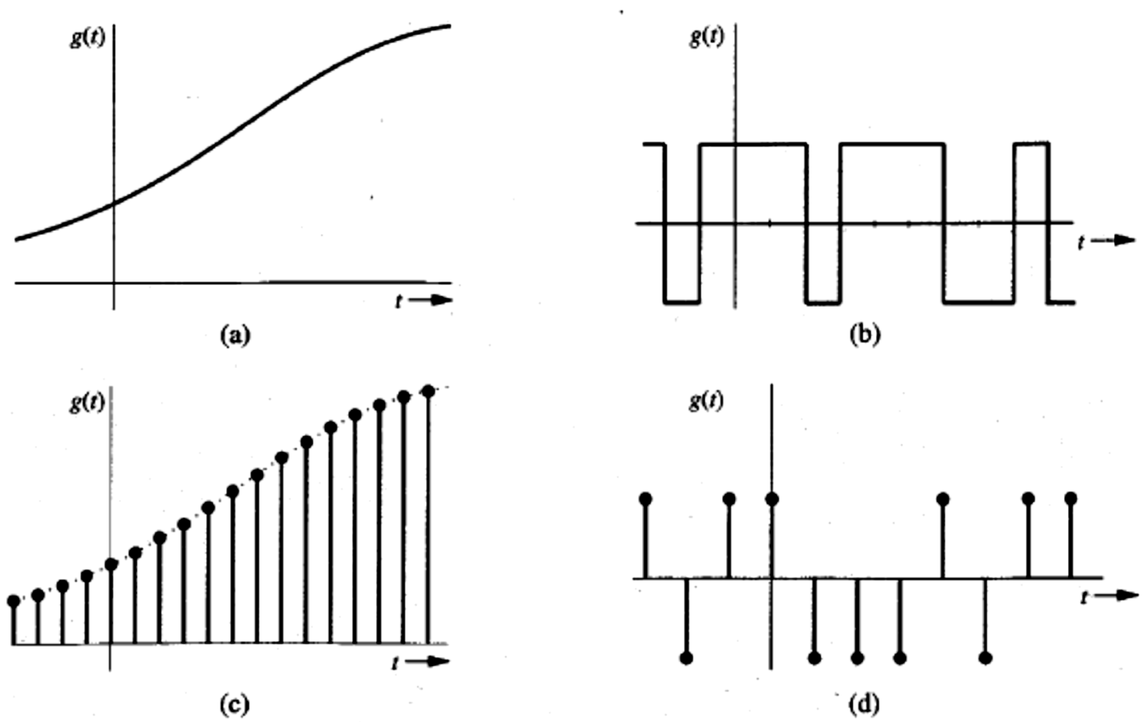
$$x(t) = t^2 + t + 1 \qquad t \in R^+$$

Digital signal:

$$x(t) = \begin{cases} 5 & for \quad 0 \leq t \leq 4 \\ 3 & for \; 4 \leq t \leq 6 \end{cases}$$



HW001: Determine is the signal are analog, digital, continuous and discrete in time?



1.2.3. Periodic and Non-periodic Signals

A signal $x(t)$ is called periodic in time if there exist a constant $T_o > 0$ such that

$$x(t) = x(t + mT_o) \qquad m \in R^+ \tag{1}$$

where $\boldsymbol{t}$ denotes time. The resultant value T_o that satisfies this condition is called the periodic of $x(t)$ .The period T_o defines the duration of one complete cycle of $x(t)$ or it is called fundamental period. A signal for which there is no value of T_o that satisfies equation (1) is called non-periodic signal.

Exercise: Determine whether or not each the following signals is periodic through using fundamental period:

1. $x(t) = \cos(w_o t)$

   Fundamental period is given as $T_o = \frac{2\pi}{w_o}$ or $T_o = \frac{1}{f}$

   Then, $x(t + T_o) = \cos(w_o(t + T_o))$

   $x(t + T_o) = \cos(w_o(t + \frac{2\pi}{w_o})) \rightarrow \ x(t + T_o) = \cos(w_o t + 2\pi))$

   $x(t + T_o) = \cos(w_o t)\ \cos(2\pi) + \sin(w_o t)\quad \sin(2\pi)$

   $where,\ \sin(2\,\pi) = 0\,, \cos(2\pi) = 1$

   $x(t + T_o) = \cos(w_o t)\ = x(t)$, hence signal is periodic

2. $x(t) = e^{jw_o t}$, use hint: $e^{jw_o t}\ = \cos(w_o t) + j\sin(w_o t)$

   Fundamental period is given as $T_o = \frac{2\pi}{w_o}$,

   Then, $x(t + T_o) = e^{jw_o(t+T_o)}\ \rightarrow\ e^{j(w_o t+2\pi)}$

   $x(t + T_o)\ = e^{jw_o t}\ e^{2\pi}\ =\ e^{jw_o t}\ (\cos(2\pi) + j\sin(2\pi)\,) = e^{jw_o t}$

   Hence, the signal is periodic because $x(t) = x(t + T_o)$

3. $x(t) = \cos\,(t + \frac{\pi}{4})$

   Fundamental period is given as $T_o = \frac{2\pi}{w_o}$, where $w_o = 1$, then $T_o = 2\pi$

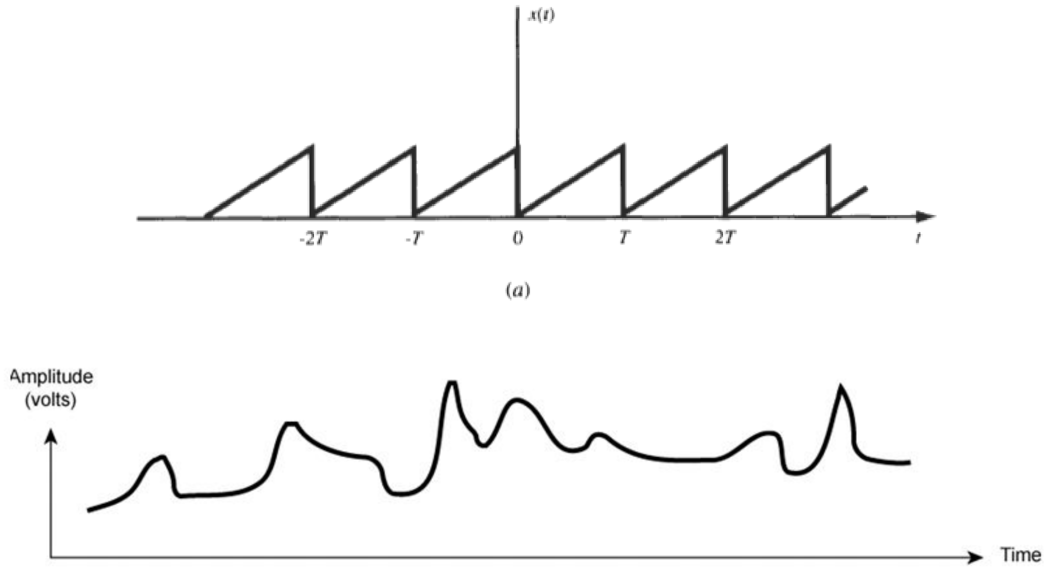
   $x(t + T_o) = \ \cos\left(t + T_o + \frac{\pi}{4}\right) \rightarrow\ \cos\left(t + 2\pi + \frac{\pi}{4}\right)$

   $x(t + T_o) = \ \cos\left(t + \frac{\pi}{4}\right)\cos\,(2\pi) + \ \sin\left(t + \frac{\pi}{4}\right)\sin\,(2\pi)$

   $x(t + T_o) = \ \cos\left(t + \frac{\pi}{4}\right) = x(t)$

HW002: Determine whether the signal is periodic or aperiodic signal?

1. $x(t) = sin\,(\frac{2\pi}{3}\ t)$ (HW)
2. $x(t) = \cos\left(\frac{\pi}{3}\ t\right) + \sin\left(\frac{\pi}{4}\ t\right)$ (HW)



(b)

Figure 1.3. (a) Periodic and (b) Non-periodic Signal.

HW003: Prove whether the signal is periodic or aperiodic signal?

(a) $x(t) = \cos\left(2t + \frac{\pi}{4}\right)$

(b) $x(t) = \cos^2 t$

(c) $x(t) = (\cos 2\pi t)u(t)$

(d) $x(t) = e^{j\pi t}$

Ans. (a) Periodic, period $= \pi$ (b) Periodic, period $= \pi$
(c) Nonperiodic (d) Periodic, period = 2

2.2.4. Energy and power signal

A signal with finite energy, is energy signal, and a signal with finite power is a power signal. In the other word, the signal with energy if the signal

$$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt < \infty \tag{2}$$

and the power content of a signal is

$$0 < \ P_x = \lim_{T \to \infty} \ \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \ < \ \infty \tag{3}$$

A signal is energy-type if $E_x < \infty$ and is power-type if $0 < Px < \infty$. A signal can*not* be both power- and energy-type because for energy-type signals $P_x = 0$ and for power-type signals $E_x = \infty$. A signal can be neither energy-type nor power-type, for example ramp signal. Another note is, every periodic signal is a power signal.

In real life all the signal generated in the lab are energy signal because power signal required infinite interval.

In fact, power is $(t) \times i(t)$, $i(t)^2 \times R$ or $\frac{v(t)^2}{R}$. Where, the $v(t)$ is represent the $x(t)$ amplitude, the absolute value used to avoid the negative part of the signal that may cancel the positive part of the signal. We have taken integral because we need to find the area under the curve between $-\frac{T}{2} \ \ to + \frac{T}{2}$, then we average the signal over the time. If the power calculated through unit ohm, then the formula above have been satisfied. The power is derived in the same way of the energy.

Exercise: Determine whether or not each the following signals is power, energy or neither:

1. $x(t) \ = e^{-at} \, u(t)$

$$E = \int_{0}^{\infty} |e^{-at}|^2 \ dt \ = \int_{0}^{\infty} |e^{-2at}| \quad dt \ = \ \frac{1}{2a} \ < \ \infty$$

Hence, it is energy signal, because it is with finite energy.

2. $x(t) = A\cos(w_o t)$ for $T_o \ interval$ and $T_o = \frac{2\pi}{w_o}$

$$x(t) = \lim_{T_o \to \infty} \frac{1}{T_o} \int_{0}^{T_o} |A\cos(w_o t)|^2 \, dt \ = \lim_{T_o \to \infty} \frac{1}{T_o} \int_{0}^{T_o} 0.5A^2 |1 + \ \cos(2w_o t)| \quad dt$$

Because signal it periodic, limit is deleted, yield:

$$x(t) = \frac{w_o}{2\pi}\int_0^{\frac{2\pi}{w_o}} 0.5A^2|\cos(2w_o t)| \quad dt \ + \frac{w_o}{2\pi}\int_0^{\frac{2\pi}{w_o}} 0.5A^2|1|^2\, dt$$

$$x(t) = \frac{w_o}{2\pi} \times (\sin 2w_o t) \ + \ \frac{w_o}{2\pi} \times (\frac{2\pi}{w_o} - 0) \times \ 0.5A^2$$

$$x(t) = \ 0.5A^2 \ \le \ \infty$$

Hence, it is power signal, because it is with finite power. Also signal is periodic.

3. $x(t) = t\,u(t)$

Signal is non periodic, hence is not power signal.

Let check whether the signal is energy signal or not:

$$E = \int_0^{\infty} |t|^2 \ dt \ = \frac{t^3}{3} \ = \ \infty - 0 \ = \ \infty$$

Hence, signal with infinite energy, it is not energy signal.

Therefore, the signal neither power nor energy.

HW 004: find whether the signal power or energy signal?

$$x(t) = \begin{cases} t & for \ \ 0 \le t \le 1 \\ 2 - t & for \ 1 \le t \le 2 \\ 0 & otherwise \end{cases}$$

HW 005: find whether the signal power or energy signal?

$$x(t) = 5\cos(\pi t) \ + \sin(5\pi t)$$

2.2.5. Deterministic and probabilistic signal

Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time ***t***.

Random signals are also called non deterministic signals are those signals that take random values at any given time and must be characterized statistically. Characterized statistically is mean first we make the experiment, then we collect the data (output event), finally we name the data to suitable function named as random variable. See figure 1.4.

Deterministic signals can be described by functions in the usual mathematical sense with time t as the independent variable. In contrast to a deterministic signal, a random signal always has some element of uncertainty associated with it, and hence it is not possible to determine its value with certainty at any given point in time.

Suppose a signal (function) change with two variables w and t, we say:

$F\ (w=5,\ t=2)$ such function is fix number defined at fix t and w

$F\ (w=5,\ t)$ such function is deterministic function with variable t and fix w

$F\ (w,\ t)$ such function is random because function does not has fix value w with time t

$F\ (w,\ t=5)$ such function is random because the amplitude is changeable with time

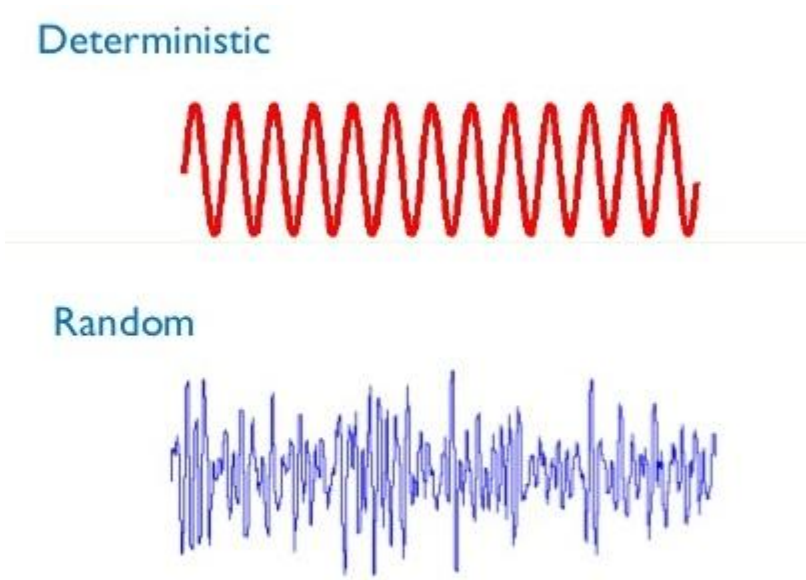


Fig. 1.3. Deterministic and Random Signal