Chapter four

Layout: **10 Hrs.**

1. Introduction.
2. Amplitude Shift Keying (ASK)
3. Phase Shift Keying (PSK).
4. Frequency Shift Keying (FSK).
5. Deferential Phase Shift Keying (DPSK).
6. **High order modulation** (M-ary Modulation or Multi-Level Modulation).
7. MPSK
8. MFSK
9. **MATLAB programs.**

Lecture One

Digital Modulation

ASK, PSK & PSK

Digital to Analog Converting (DAC)

Objective of Lecture:

Understand the way by which we convert the bit sequences into analog signals.

Behavioral goals:

- Student will be able to covert bit into analog signal.
- Differentiate between different digital modulations scheme.
- Use appropriate digital modulation technique according to given data

This lecture answer important questions which are:

What is PCM?

Why PCM is important?

How is PCM done?

Where can you exploit PCM?

What are the problems in PCM?

4.1. Introduction to Bandpass Modulation

In baseband data transmission, which we studied previously, an incoming serial data stream is represented in the form of a discrete pulse-amplitude modulated wave that can be transmitted over a baseband channel (e.g., a coaxial cable). What if the requirement is to transmit the data stream over a band-pass channel, exemplified by wireless and satellite channels? In applications of this kind, *we usually resort to the use of a modulation strategy configured around a sinusoidal carrier*. Whose**, amplitude, phase, or frequency** is varied in accordance with the information-bearing data stream.

The primary aim of the chapter is to describe some important digital band-pass modulation techniques used in practice. In particular, we describe three basic modulation schemes:

- Amplitude Shift Keying (ASK)
- Phase Shift Keying (PSK)
- Frequency Shift Keying (FSK)

4.2. Binary Amplitude Shift Keying ASK

ASK is similar to AM (amplitude modulation in analog system). But amplitude of output of the modulator can take only one of two possible amplitudes during each bit interval. The output of ASK modulator can be represented as:

$$S_{ASK} = s_{bits} \times \mathrm{A} \times \sin(2\pi f_c t) \quad for \quad 0 \le t \le T_b \tag{1}$$

Where, s_{bits} is the information message (i.e. $m(t)$) in the bit form which is 0 or 1; S_{ASK} is modulated signal, f_c is the carrier frequency or center frequency, T_b is the bit duration (Bit Peroid). In ASK, the signal is transmitted after modulation in the two possible values '1' and '0' in the bit's form, this can be achieved by making the carrier signal ON and OFF

corresponding with input of sequences of the bits. In according to input sequence values '1' and '0' in the bit's form, the ASK can be represented as:

$$S_{ASK} = \begin{cases} A \times \sin(2\pi\, f_c\, t) & for\ '1'\ \ for \quad 0 \leq\ t \leq T_b \\ 0 & for\ '0'\ \ for \quad 0 \leq\ t \leq T_b \end{cases}$$

(2)

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \leq f\ \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the 101101 is data used to be transmitted over Bandpass channel at transmission speed $50{,}000\ bps$ using ASK digital modulation. The output of ASK corresponding to digital bits:

$$S_{ASK} = \begin{cases} A \times \sin(2\pi\ 150{,}000\ t) & \rightarrow\ 1 \quad for \quad 0 \leq\ t \leq T_b \\ 0 & \rightarrow\ 0 \quad for \quad 0 \leq\ t \leq T_b \end{cases}$$

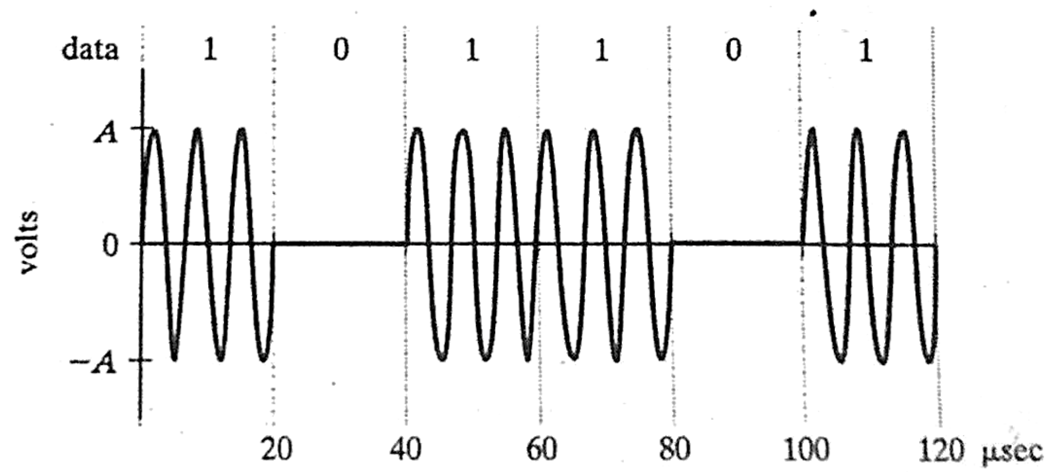


Figure: ASK modulation

Two important thing should be known to the reader, which are:

1. Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{T_s\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ bit$$

2. Bit duration in ASK modulation, it given as:

$$T_b = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

4.3. Binary Phase Shift Keying PSK

In fact, Bandpass modulation required to multiply the original data by carrier to be transmitted over wireless medium. Modulation carrier consist three variable parameters, which are amplitude, phase and frequency; changing in the amplitude in accordance to data result ASK modulation, changing in the phase in accordance to the data result PSK modulation, and finally changing in the frequency in accordance to data result FSK modulation.

Hence, in the phase modulation, carrier is switched between two phase valued in accordance to input bits. Mathematically, PSK signal represented as:

$$S_{PSK} = s_{bits} \times \mathrm{A} \times \sin(2\pi\, f_c\, t + \theta) for \quad 0 \le t \le T_b \tag{3}$$

Where, θ is the phase of the carrier. In according to input sequence values '1' and '0' in the bit's form, the ASK can be represented as:

$$S_{PSK} = \begin{cases} \mathrm{A} \times \sin(2\pi\, f_c\, t + \theta = 0^o) & \rightarrow 1\ for \quad 0 \le t \le T_b \\ \mathrm{A} \times \sin(2\pi\, f_c\, t + \theta = 180^o) & \rightarrow 0\ for \quad 0 \le t \le T_b \end{cases} \tag{4}$$

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \le f \le 250kHz$, hence the carrier frequency is the center frequency of range

$50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the 101101 is data used to be transmitted over Bandpass channel at transmission speed $50{,}000\ bps$ using PSK digital modulation. The output of PSK corresponding to digital bits:

$$S_{ASK} = \begin{cases} \text{A} \times \sin(2\pi\ 150{,}000\ t\ +\ \theta = 0^{o}) & \rightarrow\ 1\ \ for\quad 0 \leq\ t \leq T_b \\ \text{A} \times \sin(2\pi\ 150{,}000\ t\ +\ \theta = 180^{o}) & \rightarrow\ 0\ for\quad 0 \leq\ t \leq T_b \end{cases}$$

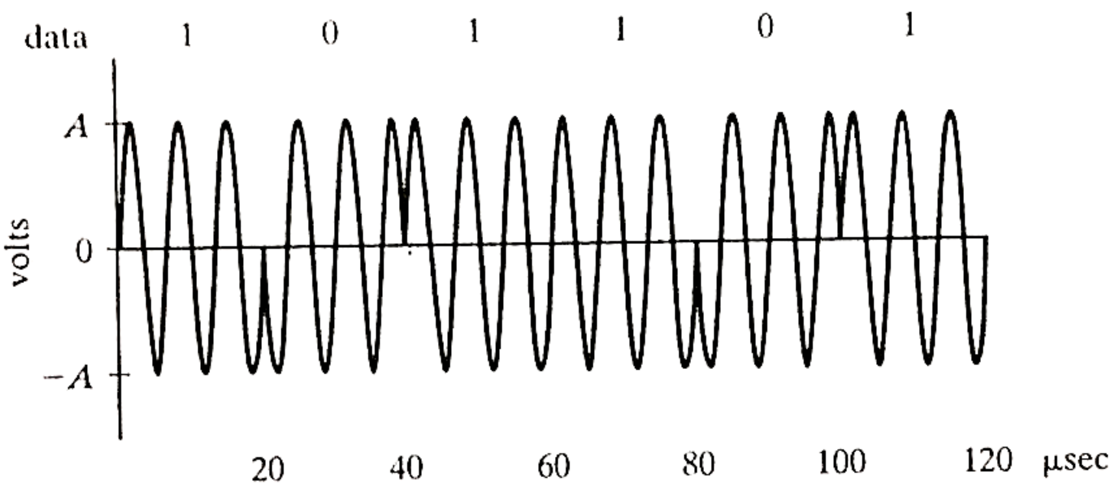


Figure: PSK modulation

Two important thing should be known to the reader, which are:

1. Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{T_s\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ bit$$

2. Bit duration in PSK modulation, it given as:

$$T_b = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

4.4. Binary Frequency Shift Keying PSK

Frequency shift keying uses changes in the frequency of the carrier signal to transmit bit form information over carrier. FSK can described as follows:

$$S_{FSK} = \begin{cases} \mathrm{A} \times \sin(2\pi\,(f_c + \Delta_c)\,t\,) & \rightarrow 1 \quad for \quad 0 \leq t \leq T_b \\ \mathrm{A} \times \sin(2\pi\,(f_c - \Delta_c)_c\,t\,) & \rightarrow 0 \quad for \quad 0 \leq t \leq T_b \end{cases} \tag{5}$$

Where, Δ_c is frequency offset. With FSK, the carrier is shifted up in the frequency by Δ_c to signify '1' and is shifted down in frequency by Δ_c to signify a '0'.

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \leq f \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the 101101 is data used to be transmitted over Bandpass channel at transmission speed $50{,}000\ bps$ using FSK digital modulation with frequency offset $\Delta_c = 50{,}000\ kHz$. The output of PSK corresponding to digital bits:

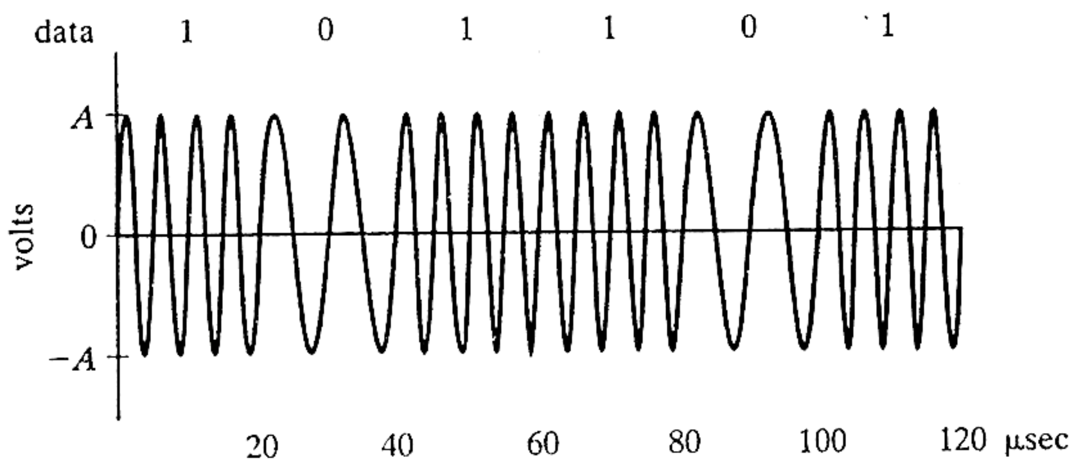


Figure: PSK modulation

$$S_{ASK} = \begin{cases} \text{A} \times \sin(2\pi\ (150{,}000 + 50{,}000)\ t\ +\ \theta = 0^o) & \rightarrow\ 1\ \ for\ \ \ 0 \leq\ t \leq T_b \\ \text{A} \times \sin(2\pi\ (150{,}000 + 50{,}000)\ t\ +\ \theta = 180^o) & \rightarrow\ 0\ \ for\ \ \ 0 \leq\ t \leq T_b \end{cases}$$

Two important thing should be known to the reader, which are:

1. Number of cycle within 1-bit period (bit duration), it is given as:

$$No.of\ Cycle = \frac{f_c +\ \Delta_c}{T_s\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{200{,}000}{50{,}000} = 4\ cycle\ per\ bit$$

2. Number of cycle within 0-bit period (bit duration), it is given as:

$$No.of\ Cycle = \frac{f_c -\ \Delta_c}{T_s\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{100{,}000}{50{,}000} = 2\ cycle\ per\ bit$$

3. Bit duration in ASK modulation, it given as:

$$T_b = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

HW: Consider a bandpass digital communication system using a carrier frequency of 600 kHz and a transmission speed of 100,000 bits/sec.

a. Draw the waveform corresponding to the data sequence "1101001" if ASK is chosen as the modulation technique.
b. Draw the waveform corresponding to the data sequence "1101001" if PSK is chosen as the modulation technique.
c. Draw the waveform corresponding to the data sequence "1101001" if FSK is chosen as the modulation technique and a frequency offset of 100 kHz is used.

4.5. deferential Phase Shift Keying DPSK (Non Coherent)

With DPSK, the data are encoded by means of changes in phase rather than by absolute value of phase in the carrier. For binary PSK representation of binary bits as follow:

- **1** can be encoded as no change in the phase as used for previous bit.
- **0** can be encoded as 180^0 change in the phase relative to previous bit.

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \leq f \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the 101101 is data used to be transmitted over Bandpass channel at transmission speed $50{,}000\ bps$ using PSK digital modulation. The output of PSK corresponding to digital bits:

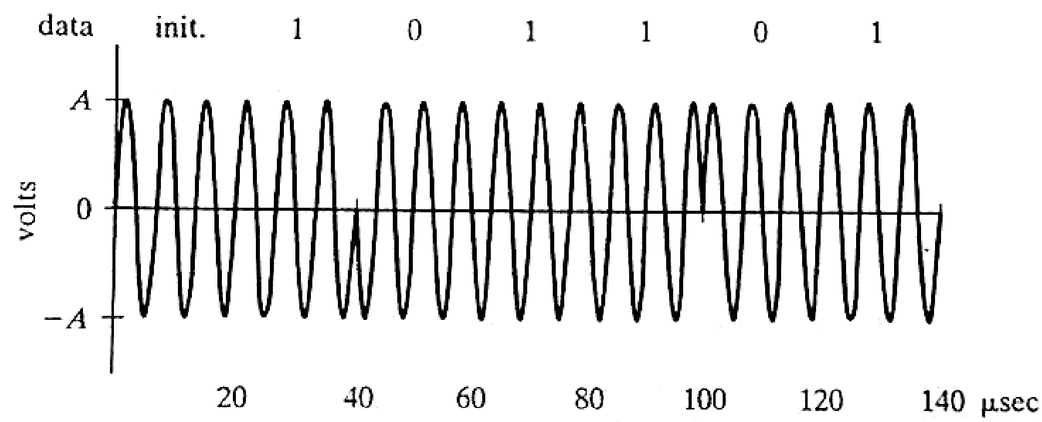


Figure: DPSK

Two important thing should be known to the reader, which are:

1. Number of cycle within bit period (bit duration), and it is given as:

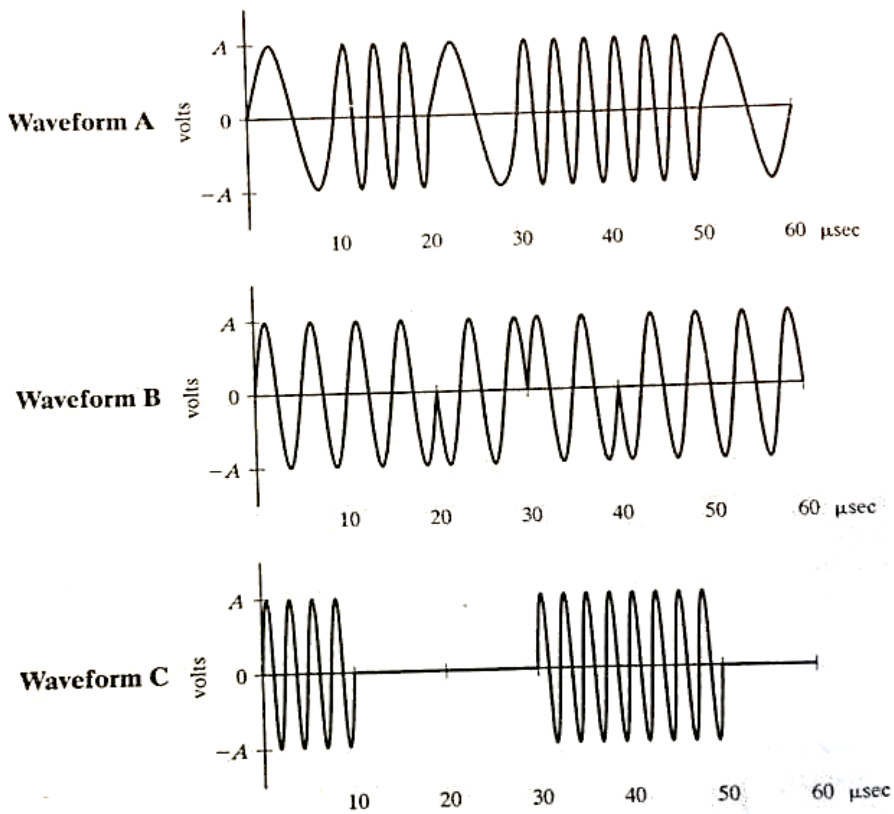
$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{T_s\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ bit$$

2. Bit duration in PSK modulation, it given as:

$$T_b = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

HW: See each waveform given in the figure below and answer the following:



Find:

1. Data rate (Transmission Speed)
2. Determine the modulation technique of each wave
3. Determine the carrier frequency
4. The transmitted data
5. Offset frequency, where applicable

Q: *Why* modulate the signal to medium frequencies, but we have to be careful not goes for very high frequencies and not down to low frequencies?

4.6. High Order Modulation (M-ary Modulation or Multi-Level Modulation)

In recent year, we have been trying to obtain more and more communication services out of limited amount of the spectrum as result, channelization in the various system is becoming bandlimited. Sampling process is another issue, where sampling frequency choosing at least two times of signal frequency.

In any attempt to improve this scarcity, high order modulation method has been used which offer greater bandwidth efficiency. The improvement in the bandwidth efficiency come from allocating more bits per signal carrier frequency or greater bits per symbol (level). It clears the advantage of high order modulation can be increased the **bandwidth efficiency** compared to binary digital modulation technique. Disadvantage of M-ary modulations is that they are more complex and more susceptible to noise than compare to binary modulation technique.

In the binary digital modulation number of bits per symbol (carrier) is $n = 1$, it mean number of distinct symbol is $M = 2^1 = 2$ symbols rather than high order modulation, number of bits $n \geq 2$ which mean number of symbols are $M = 2^n$. The relation between number of bits and number of distinct symbols are given as:

$$n = \log_2 M \quad and \quad M = 2^n \tag{6}$$

Transmission speed (data rate or transmission rate) using high order modulation become n times of binary modulation, where transmission speed in bit per second expressed as

$$Transmission\ speed\ in\ bps\ = n \times\ transmission\ speed\ in\ \frac{symbol}{s} \tag{7}$$

4.6.1. M-ary Phase Shift Keying

The M-ary PSK modulation allocate more than single bit to each carrier symbol uses different phase at each carrier symbol. The difference between each carrier symbol is determined by

$$d = \frac{360^o}{M}$$

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $50kHz \leq f \leq 250kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 50 100 **150** 200 250). Assume the **1101100001** is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000}\ \boldsymbol{symbol/sec}$ using 4PSK digital modulation. Find transmission speed in bps?

- first we determine number of distinct level which is M = 4
- Then we find number of bits per each symbol, $n = \log_2 M \ \rightarrow n = \log_2 2^2 \ \rightarrow n = 2\ bits$.
- Now we find distance between each phase of symbols, which is given as

$$d = \frac{360^o}{M} = \frac{360}{4} = 90^o$$

- We allocate each two bits to distinct symbol as follow:

$$00 \ \rightarrow \quad A\sin(2\pi f_c t + \emptyset_0 = 45^0)$$
$$00 \ \rightarrow \quad A\sin(2\pi 150{,}000 t + \emptyset_0 = 45^0)$$
$$01 \ \rightarrow \quad A\sin(2\pi f_c t + \emptyset_1 = 135^0)$$
$$01 \ \rightarrow \quad A\sin(2\pi 150{,}000 t + \emptyset_1 = 135^0)$$
$$10 \ \rightarrow \quad A\sin(2\pi f_c t + \emptyset_2 = 225^0)$$
$$10 \ \rightarrow \quad A\sin(2\pi 150{,}000 t + \emptyset_2 = 225^0)$$
$$11 \ \rightarrow \quad A\sin(2\pi f_c t + \emptyset_3 = 315^0)$$
$$11 \rightarrow A\sin(2\pi 150{,}000_c t + \emptyset_3 = 315^0)$$

- Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ symbol$$

- Bit duration in PSK modulation, it given as:

$$T_{symbol} = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

- Transmission speed in bps = $n \times$ transmission speed in $\frac{symbol}{s}$

  Transmission speed in bps = $2 \times\ 50{,}000\ \frac{symbol}{s} = 100{,}000\ bps$

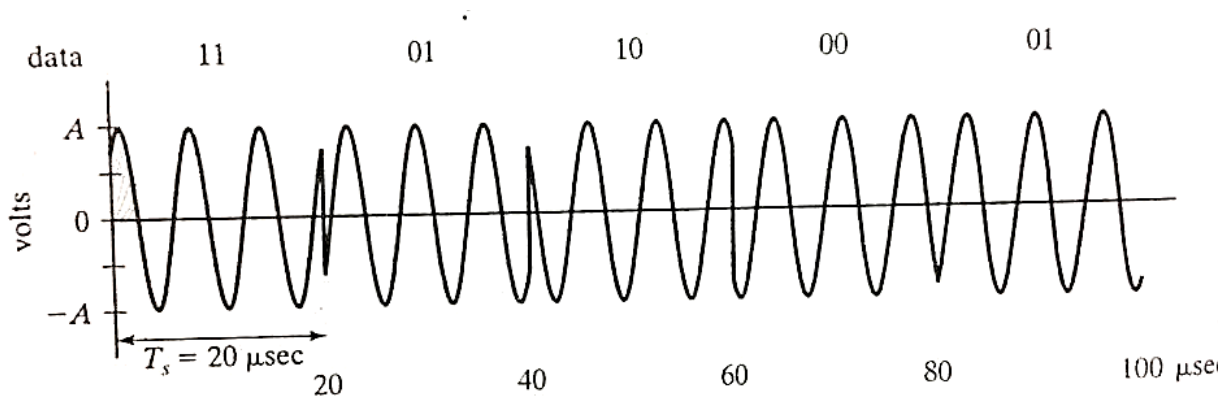


Figure: 4PSK representation

4.6.2. M-ary Frequency Shift Keying

The M-ary FSK modulation allocate more than single bit to each carrier symbol uses different frequency at each carrier symbol. In his case, we can create different symbol with different frequencies. The symbols divided into two groups; first group of symbols (carrier) are frequencies up and second group of symbols are down. Carrier frequencies is given as:

$$up\ frquencies\ \{f_c + \Delta_c, f_c + 2\Delta_c, f_c + 3\Delta_c, f_c + \frac{M}{2}\Delta_c$$

$$down\ frquencies\ \{f_c - \Delta_c, f_c - 2\Delta_c, f_c - 3\Delta_c, f_c - \frac{M}{2}\Delta_c \qquad (8)$$

Now let justify our discussion with example, suppose a channel can be pass frequencies in range $75kHz \leq f \leq 275kHz$, hence the carrier frequency is the center frequency of range $50\ kHz\ to\ 250\ kHz$ (i.e. 75 ← **175** → 275). Assume the **1101100001** is data used to be transmitted over Bandpass channel at transmission speed $\mathbf{50,000}\ \boldsymbol{symbol/sec}$ using 4PSK digital modulation, frequency offset $\Delta_c = 25\ kHz$. Find transmission speed in bps?

- first we determine number of distinct level which is M = 4
- Then we find number of bits per each symbol, $n = \log_2 M \ \rightarrow n = \log_2 2^2 \ \rightarrow n = 2\ bits$.
- Carrier frequencies of four groups are given as:

$$up\ frquencies\ \{200000, 225000\}$$

$$down\ frquencies\ \{150000, 125000\}$$

$$00 \rightarrow\ A\sin(2\pi(f_c +\ \Delta_c)t)$$

$$00 \rightarrow\ A\sin(2\pi 200000t)$$

$$01 \rightarrow A\sin(2\pi f(f_c +\ 2\Delta_c)_c t)$$

$$01\ \rightarrow\ A\sin(2\pi 225{,}000t)$$

$$10 \rightarrow\ A\sin(2\pi(f_c -\ \Delta_c)t)$$

$$10 \rightarrow\ A\sin(2\pi 150{,}000t)$$

$$11 \rightarrow A\sin(2\pi(f_c -\ 2\Delta_c)t)$$

$$11 \rightarrow\ A\sin(2\pi 125{,}000_c t)$$

- Number of cycle within bit period (bit duration), and it is given as:

$$No.of\ Cycle\ of\ \mathbf{00} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{200{,}000}{50{,}000} = 4\ cycle\ per\ symbol$$

$$No.of\ Cycle\ of\ \mathbf{01} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{225{,}000}{50{,}000} = 4.5\ cycle\ per\ symbol$$

$$No.of\ Cycle\ of\ \mathbf{10} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{150{,}000}{50{,}000} = 3\ cycle\ per\ symbol$$

$$No.of\ Cycle\ of\ \mathbf{11} = \frac{f_c\ (carrier\ frequency)}{TS\ (transmission\ speed)}$$

$$No.of\ Cycle = \frac{125{,}000}{50{,}000} = 2.5\ cycle\ per\ symbol$$

- Bit duration in PSK modulation, it given as:

$$T_{symbol} = \frac{1}{T_s} = \frac{1}{50{,}000} = 20\ \mu s$$

- Transmission speed in bps = n × transmission speed in $\frac{symbol}{s}$

  Transmission speed in bps = 2 × $50{,}000\ \frac{symbol}{s} = 100{,}000\ bps$

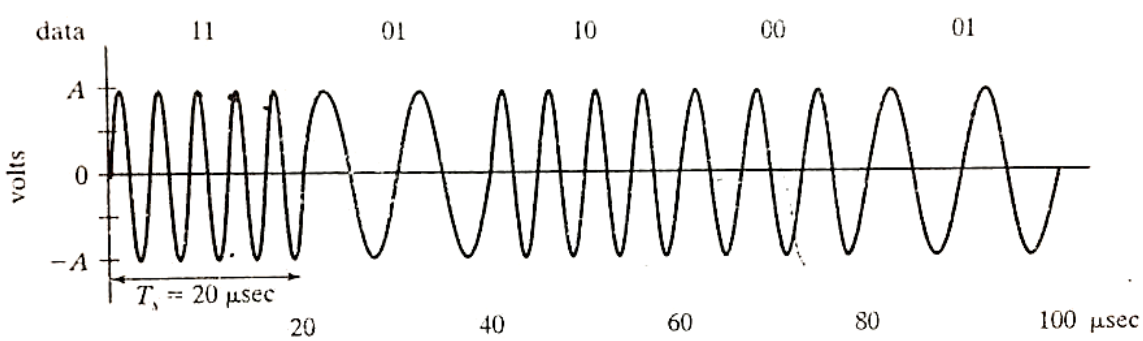


Figure: 4FSK representation